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Modeling of non-proportional cyclic loading with a simple yield surface distortion model

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ABSTRACT

In this study, a yield surface distortion model coupled with nonlinear kinematic hardening model is proposed. The new yield surface distortion model is used to simulate proportional and non-proportional cyclic loading paths. Distortion of the yield surface and its nonlinear effects on the plastic flow have been studied by researchers and various models. However, these models have been rarely applied to different non-proportional cyclic loading paths. The yield surface evolution is obtained by the new model in cyclic loading and its results are compared with the experimental results of Dannemeyer [30]. The new model with few constants predicts well the yield surface evolution under non-proportional loading paths which leads to well comparison of the numerical results with respect to the experimental results.

1. Introduction

In order to better model the plastic flow, cyclic hardening or cyclic softening, some theoretical models were developed. Modeling of the material behavior under non-proportional loading paths is one of the main concerns. Anisotropic hardening and the yield surface distortion models [1–6] have been proposed by researchers and the proportional loading paths were used to verify them. These models usually consist of many material constants. As an example, the anisotropic hardening model of [2] consists of 11 material constants and the proportional loading for a constant and proportional variation of was considered to obtain yield surface deformation in space. The shape of the yield surface which was characterized by the model of [2], showed the main phenomena of the yield surface distortion. Based on the thermodynamics of dissipative materials, another model of the yield surface distortion was proposed in [5]. The results of this model were compared with the results of different experimental proportional loading paths. The convexity of the yield surface model is an essential subject in yield surface function [6].

Non-proportional loading effects were studied by few researchers [7–14]. For example, Cailletaud et al. [9] showed that with increasing the degree of multiaxiality, hardening increased. Tanaka et al. [10] observed experimentally that hardening is dependent on the plastic strain paths. These paths consisted of strain control, stress control or combined stress–strain control paths. Different nonlinear kinematic hardening models, with many material constants, were proposed to predict stress–strain behavior under non-proportional cyclic loading (see [15,16]). Some of the researchers showed that these models can predict the material behavior under some of the non-proportional loading paths but they failed to predict other cyclic loading paths. For example, Abdel-Karim [17] studied the responses of many nonlinear kinematic hardening rules under various stress controlled histories which had been obtained experimentally by Hassan and Taleb [18]. It was shown that none of the studied models could predict ratcheting response of the cyclic loading experiments. Kang et al. [19] studied the uniaxial and non-proportionally multiaxial ratcheting behaviors by a cyclic constitutive model of plasticity. Bari and Hassan [20] and Rahman et al. [21] studied different non-proportional multiaxial loading. They concluded that the yield surface shape evolution should be considered in the study of the multiaxial loading ratcheting.

Some of the authors obtained experimentally the yield surface evolution during plastic deformation [22–29]. The yield surface shape was obtained under different uniaxial tension, torsional or proportional loadings in these studies. To our knowledge, only Dannemeyer [30] obtained yield surface evolutions under three different non-proportional multiaxial cyclic loading paths.

In this study, the evolution of the yield surface form under different non-proportional loading paths, given by Dannemeyer [30], was modeled by a new simple yield distortion model and compared

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with the experimental results of [30]. Cyclic stress–strain response was also modeled and compared with the mentioned experimental results.

2. Main phenomena of subsequent yield surface

The way Subsequent yield surface from has been studied by many researchers. It was found that the yield surface distorts, rotates and depending on the studied materials, has a cross effect. The detailed description of the subsequent yield surface form has been discussed by Dannemeyer [30]. The majority of the subsequent yield surface study has been done under proportional loading experiments and only some authors studied the yield surface evolution during non-proportional loading. Naghdi et al. [23] were among the first researchers who obtained the yield surface evolution under biaxial tension-torsion proportional loading. They found that the subsequent yield surface is not the blown up of the initial yield surface. Then, it was found that the yield surface has a nose shape in the loading and a flat shape in the reverse loading directions. It rotates, and the nose shape is more dominant in tension than in torsion. Cross effect is negligible in yield surface distortion for some of the studied metals [24–28].

In this study, the subsequent yield surface evolutions which were obtained experimentally by Dannemeyer [30] were modeled by a simple yield surface distortion model. The loading paths used in [30] were the proportional and non-proportional cyclic loading paths. The test specimen was tubular specimen of mild steel Fe 510. Loading paths consisted of the tensile and torsional strain paths. Yield surface evolutions under different non-proportional loading paths were obtained experimentally. Stress–strain hysteresis loops of the saturated material were also obtained experimentally by Dannemeyer [30]. The results of the proposed model were also compared with the results of the Kowalski et al. [31] model.

3. Yield surface distortion model

Based on the normality rule, the plastic strain increment is related to the normal unit tensor direction of the yield surface. So, correct determination of the plastic strain increment is dependent on the correct prediction of the yield surface form, especially in non-proportional loadings [32]. Baltov and Sawczuk [33] proposed a distortional hardening model based on the von-Mises yield surface. Rokhgireh and Nayebi [32] applied the yield surface model of Baltov and Sawczuk to the different cyclic loading paths and better prediction of ratcheting strains of different steels was obtained. Baltov and Sawczuk added the exterior product of the plastic strain tensor, ε^p_{ij} and A_{ijkl} to the forth order isotropic tensor, I_{ijkl}, and the von-Mises yield surface was modified to (Eq. (1)):

$$F = N_{ijkl}(S_{ij} - \alpha'_{ij})\ \ (S_{kl} - \alpha'_{kl}) - \frac{2}{3}\sigma_y^2 \tag{1}$$

α'_{kl}, S_{ij}, σ_y are deviatoric back stress, deviatoric stress and yield stress, respectively. The 4th order tensor N_{ijkl} is defined by Eq. (2).

$$\begin{cases} N_{ijkl} = I_{ijkl} + A_{ijkl} \\ I_{ijkl} = \dfrac{1}{2}\left[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj} - \dfrac{2}{3}\,\delta_{ij}\delta_{kl}\right] \\ A_{ijkl} = A_0\varepsilon^p_{ij}\varepsilon^p_{kl} \end{cases} \tag{2}$$

I_{ijkl} is 4th order isotropic tensor. The second part of N_{ijkl} is an anisotropic tensor and causes the anisotropic behavior of the yield surface in the stress space.

It was shown experimentally that the yield surface has a nose shape (inflate) in the loading and a flat shape (deflate) in the reverse loading direction [22–24]. But, Baltov and Sawczuk's yield surface model cannot simulate the yield surface distortion properly, and it expands or contracts in both loading and opposite loading direction in the stress space (Fig. 1). Their yield surface model can predict the rotation of the yield surface (parameter μ) and it does not represent any cross effect. In order to predict the nosed and flattened regions of the yield surface, the angle θ is defined by the angle between tensors $S_{ij} - \alpha'_{ij}$ and α'_{ij}. Nosed shape occurs experimentally in $\theta = 0$ (loading direction) and flattened region occurs in $\theta = \pi$ (opposite of the loading direction). But, the Baltov and Sawczuk's model predicts nosed shape occurring in both $\theta = 0$ and $\theta = \pi$ when $A_0 < 0$ and flattened regions occur in both $\theta = 0$ and $\theta = \pi$, when $A_0 > 0$. The main disadvantage of their yield surface model is the lack of flattened region in opposite loading direction when $A_0 < 0$. In order to predict the yield surface distortion realistically, the continuous change of A_0 sign from negative in $\theta = 0$ to positive in $\theta = \pi$ is necessary which causes the nosed shape in loading direction and flattened region in opposite of the loading direction. Continuous change of the sign of A_0 is achieved by using a triangular function $\cos(\theta)$. So, a new directional hardening model was presented by Rokhgireh and Nayebi [34]. Eq. (3) shows the proposed yield surface model.

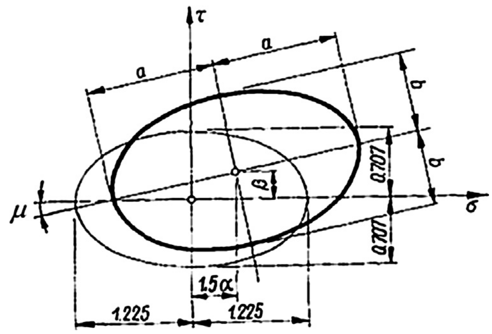

Fig. 1. Yield surface evolution according to the Baltov and Sawczuk's yield surface model [33] with a negative A_0 constant.

$$F(\sigma, \alpha', \varepsilon^p) = (S_{mn} - \alpha'_{mn})(S_{mn} - \alpha'_{mn}) + A_0\cos(\theta)\,\varepsilon^p_{kl}\varepsilon^p_{ef}(S_{kl} - \alpha'_{kl})(S_{ef} - \alpha'_{ef}) - \frac{2}{3}\sigma_y^2 \tag{3}$$

where

$$\begin{cases} \cos(\theta) = \dfrac{(S_{pq} - \alpha'_{pq})\,\alpha'_{pq}}{\left\|S - \alpha'\right\|\ \|\alpha'\|} \\ \sigma_y = \sigma_y^0 - C\varepsilon^p_{eq}\cos^2(\theta) \end{cases} \tag{4}$$

and

$$\varepsilon^p_{eq} = \int\sqrt{\frac{2}{3}\dot{\varepsilon}^p_{mn}\,\dot{\varepsilon}^p_{mn}dt} \tag{5}$$

S, α', ε^p and σ_y are deviatoric stress, deviatoric back stress, plastic strain tensors and yield stress, respectively. Similar idea of Eq. (4), which takes into account the trace of product of back stress with the direction of the radius of the yield surface, was given by Feigenbaum and Dafalias [5] and Ortiz and Popov [35]. In order to limit the expansion of the yield surface in loading and reverse loading directions, the term of $\cos^2(\theta)$ was added to the yield stress. Furthermore, this term vanishes the cross effect. σ_y^0 and C are the initial yield stress and a material constant, respectively. This model is capable to predict the nose and flattened regions in the loading direction and the opposite direction, respectively. Moreover, the rotation of the yield surface in the stress space could be modeled by the proposed yield surface distortion model.