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Affine arithmetic applied to transient statistical energy analysis of a two-oscillator system

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ABSTRACT

This paper applies affine arithmetic to transient statistical energy analysis (SEA) of a two-oscillator system, and the influence of the measurement errors of parameters on predicted transient energy is revealed. By considering the damping loss factors and coupling loss factors with measurement errors as interval variables, the mathematical expressions of the total energy interval and the peak energy interval can finally be derived. Then two flat plates which are perpendicular to each other and joined together through bolts are exploited as numerical example to demonstrate the feasibility and effectiveness of the presented approach. Meanwhile, the structural transient energy calculated from the presented approach is compared with that obtained from the traditional method which did not consider the measurement errors of damping loss factors and coupling loss factors.

1. Introduction

Using the steady-state SEA, the coupling dynamics problems in high frequency can be addressed well. However, the steady-state SEA formalism is not directly applicable to the analysis of transient vibrations because the total energy of each subsystem changes over time in the transient SEA. The input power is the sum of the output power and the derivative of energy with respect to time. Furthermore, the time-varying total energy of each subsystem can be obtained by solving a set of simultaneous energy balance equations.

Over the past several decades, a great many theoretical innovations in transient SEA have been proposed and some successful applications have been reported, which greatly enriched and extended the contents of transient SEA. Yamazaki et al. [1] discussed the characteristics of prediction by transient SEA for a two-subsystem system, focusing on the relationship between the predictions and SEA loss factors. Robinson and Hopkins [2] used experimental statistical energy analysis to determine the steady-state coupling loss factors, and the measured coupling loss factors were then incorporated in a two-subsystem transient SEA model for comparison with measured maximum sound pressure levels and the maximum vibration levels. Good agreement was achieved between measurements and predictions. Mao et al. [3,4] introduced the transient SEA method into the identification of impact load.

According to the transient SEA, Pinnington and Lednik [5] provided the exact transient energy response of a two-oscillator system subject to an impulse excitation. Meanwhile, the peak energy was also investigated. The exact transient energy response calculated by Pinnington and Lednik is feasible and effective, but the measurement errors of damping loss factor and coupling loss factor of each subsystem were not considered in their research. In the real situation, the damping loss factors and the coupling loss factors are usually very small parameters in SEA, so it is difficult to accurately measure these parameters. Sometimes the measurement errors of damping loss factors and coupling loss factors are big and non-ignorable. Therefore, it is worth exploring the effect of the measurement errors of parameters on the predicted structural energy.

At present, interval analysis methods are well recognized as a powerful tool for dealing with the problem of measurement errors because interval analysis methods require a small amount of information and can effectively reduce the influence of human factor. Under the assumption of small variations about the nominal parameter value, Qiu et al. [6,7] made use of a first-order interval perturbation approach to determine the influence of interval parameters on static displacement of structures. Yang et al. [8] proposed an interval finite element method for the frequency response function of structures with uncertain parameters. Affine arithmetic was developed as an enhancement of the basic interval arithmetic. Manson [9] applied affine arithmetic to uncertainty modeling in structural analysis. Miyajima and Kashiwagi [10] conducted a theoretical study on the existence of solution in nonlinear systems by

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using affine arithmetic. The dynamic eigenvalue analysis of structures with interval parameters based on affine arithmetic has been reported by Zhu and Chen [11]. Degrauwe et al. [12] suggested a novel method to solve affine systems with linear equations, which allowed for the application of affine arithmetic in finite element analysis. Staudt et al. [13] developed an interval Newton method based on the modified affine arithmetic to find all possible stationary points of the tangent plane distance function.

This paper applies affine arithmetic to transient SEA of a two-oscillator system, and the influence of the measurement errors of parameters on predicted transient energy is revealed. By considering the damping loss factors and coupling loss factors with measurement errors as interval variables, the total energy interval and the peak energy interval can be provided. The whole content is organized as follows. In Section 2, the basic SEA and interval theory are described in detail. In Section 3, mathematical expressions of the total energy interval and the peak energy interval are derived. In Section 4, a two-plate system is used as numerical example to demonstrate the feasibility and effectiveness of the presented approach, where the presented approach is compared with the method proposed by Pinnington and Lednik [5]. In Section 5, some useful conclusions are given.

2. Basic SEA and interval theory

2.1. SEA theory

The dissipated power of subsystem i in SEA is evaluated by

$$p_{id} = \omega\eta_i E_i \tag{1}$$

where ω is the center frequency in the band $\Delta\omega$, η_i is the damping loss factor which represents the rate of energy transfer out of the subsystem i into an unrecoverable energy form (such as heat), and E_i is the total energy of subsystem modes at frequency ω.

The absolute power flow from subsystem i to subsystem j is

$$p_{ij} = \omega\eta_{ij}E_i - \omega\eta_{ji}E_j \tag{2}$$

where η_{ij} is the coupling loss factor which represents the rate of energy transfer out of subsystem i into subsystem j. Assuming the number of the subsystems is k, the power balance equation of subsystem i can be written as [14]:

$$p_{i,in} = \dot{E}_i + \omega\eta_i E_i + \sum_{j=1, j\neq i}^{k} (\omega\eta_{ij}E_i - \omega\eta_{ji}E_j) \tag{3}$$

where $p_{i,in}$ is the input power from the external source of excitation.

2.2. Interval theory

In interval analysis, an uncertain variable is represented by a closed and finite interval. An interval variable x^I is fully characterized by its lower bound x^l and its upper bound x^u

$$x^I = \{x | x^l \le x \le x^u\} = [x^l, x^u] \tag{4}$$

The interval median is $x^c = (x^l + x^u)/2$, while the interval dispersion is $x^r = (x^u - x^l)/2$. The basic operators addition (+), subtraction (−), multiplication (×) and division (/) are generalized for the case of interval variables as: [15]

$$\begin{aligned} x^I + y^I &= [x^l + y^l, x^u + y^u] \\ x^I - y^I &= [x^l - y^u, x^u - y^l] \\ x^I \times y^I &= [\min\{x^l y^l, x^l y^u, x^u y^l, x^u y^u\}, \max\{x^l y^l, x^l y^u, x^u y^l, x^u y^u\}] \\ x^I / y^I &= x^I \times [1/y^u, 1/y^l] \end{aligned} \tag{5}$$

It should be pointed out that Eq. (5) has usually the problem of interval extension. Take a function $f(x) = (1 + x)/x$ and an interval variable $x^I = [1, 2]$ as an example, the evaluation of $f(x^I)$ can be obtained by Eq. (5)

$$y_1^I = f(x^I) = (1 + [1, 2])/[1, 2] = [1, 3] \tag{6}$$

When the function $f(x)$ is simplified to $1 + (1/x)$ prior to the evaluation of $f(x^I)$, however, the exact result is found:

$$y_2^I = f(x^I) = 1 + 1/[1, 2] = [3/2, 2] \tag{7}$$

The main reason of interval extension is that x^I is used more than one time. Manson [9] provided the affine arithmetic which can improve the problem of interval extension to some extent. In affine arithmetic, $x^I = [x^l, x^u]$ and $y^I = [y^l, y^u]$ are expressed as:

$$x^I = x_0 + x_1[\varepsilon_1] + \cdots + x_n[\varepsilon_n] + x_e[\varepsilon_e] \tag{8}$$

$$y^I = y_0 + y_1[\varepsilon_1] + \cdots + y_n[\varepsilon_n] + y_e[\varepsilon_e] \tag{9}$$

where $[\varepsilon_i] = [-1, 1]\,(i = 1, \ldots, n)$ and $[\varepsilon_e] = [-1, 1]$. The basic operations are

$$x^I + y^I = (x_0 + y_0) + \sum_{i=1}^{n}(x_i + y_i)[\varepsilon_i] + (x_e + y_e)[\varepsilon_e] \tag{10}$$

$$x^I - y^I = (x_0 - y_0) + \sum_{i=1}^{n}(x_i - y_i)[\varepsilon_i] + (x_e + y_e)[\varepsilon_e] \tag{11}$$

$$\begin{aligned} x^I \times y^I =\ & x_0 y_0 + \frac{1}{2}\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{n}(x_0 y_i + x_i y_0)[\varepsilon_i] \\ & + \left(\sum_{i=1}^{n}(|x_i|y_e + x_e|y_i|) + |x_0|y_e + x_e|y_0| + x_e y_e \right. \\ & \left. + \frac{1}{2}\sum_{i=1}^{n}|x_i y_i| + \sum_{i=1}^{n}\sum_{j=i+1}^{n}|x_i y_j + x_j y_i| \right)[\varepsilon_e] \end{aligned} \tag{12}$$

$$\begin{aligned} 1/x^I =\ & -\frac{x_0}{[x]^-[x]^+} + \frac{1}{2[x]^-} + \frac{1}{2[x]^+} \\ & + \frac{1}{\sqrt{[x]^-[x]^+}} + \sum_{i=1}^{n} -\frac{x_i}{[x]^-[x]^+}[\varepsilon_i] \\ & + \left(-\frac{x_e}{[x]^-[x]^+} + \left| \frac{1}{2[x]^-} + \frac{1}{2[x]^+} - \frac{1}{\sqrt{[x]^-[x]^+}} \right| \right)[\varepsilon_e] \end{aligned} \tag{13}$$

in which $[x]^- = x_0 - \sum|x_i| - x_e$ and $[x]^+ = x_0 + \sum|x_i| + x_e$

Consider the function $f(x) = (1 + x)/x$ and the interval variable $x^I = [1, 2]$ again. Setting $x^I = 3/2 + [\varepsilon_1]/2$ and using Eqs. (12) and (13), we have

$$f(x^I) = 1.705 - 0.271[\varepsilon_1] + 0.191[\varepsilon_e] = [1.243, 2.168] \tag{14}$$

It can be found that $f(x^I)$ calculated by affine arithmetic is closer to the exact result.

3. Affine arithmetic for transient SEA

The subsystems in SEA are groups of similar modes within the physical components of a system. The physical components of a system are often easily identified as the relatively uniform sections of