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Wave analysis and control of double cascade-connected damped mass-spring systems

K. Nagase*

Department of Opto-Mechatronics, Wakayama University, 640-8510 Wakayama, Japan

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ABSTRACT

This paper presents wave analysis and control for double cascade-connected damped mass-spring systems, whose mass is connected beyond the adjacent masses. The system is motivated by a cantilevered tensegrity beam supporting tensile and compressive forces. The wave solution is derived from a recurrent formula, and the properties of the propagation constants are precisely investigated. Elimination of reflected waves provides the impedance matching controller. We show that the impedance matching controller can be constructed from a similarity transformation of the characteristic impedance matrix by a matrix composed of the propagation constants. A numerical example of vibration control of a tensegrity beam is shown.

1. Introduction

Demands for mechanical structures to be lighter, faster, and energy efficient have meant active vibration control of flexible structures has been attracting increasing attention in recent years. Most active vibration control designs are based on modal analysis (modal control) (see, for example, [4,25,31]), a well-established technique. However, modal control encounters difficulties in controlling modally dense structures (i.e., very flexible structures or large scale structures), because modal frequencies and modal shapes are extremely sensitive to modeling errors [27,19].

For certain types of flexible structures composed of simple members, such as vibrating strings, beams, flexural waveguides, etc., dynamical response can be described by wave motion. In wave analysis, system dynamics are described by transfer functions called secondary constants (propagation constants and characteristic impedances). Secondary constants are independent of the size or length of the structure, and are less sensitive to modelling errors [27,19,36]. Therefore, control design based on wave analysis (wave control) is expected to be efficient for modally dense structures.

Wave control of flexible structures was originally developed for lateral beam vibration by [46], and extended to junction control of structural networks [9,8,27,28]. As a fundamental element in structures, wave properties of slender beams have been the topic of much research [18,10,7,23,45]). Wave control of a rectangular panel was investigated in [15,16], and of cascade connected mass-spring systems in [48,36]. Hybrid wave/mode control was proposed in [24,22], and wave propagation under periodic discontinuities on a uniform base was investigated in [6,26,40]. In contrast to these model based approaches, some researches have utilized the input/output response of the real structures to estimate their secondary constants [19,21,32].

In this paper, we develop wave analysis and wave control for double cascade-connected damped mass-spring systems, as an extension of single cascade-connected damped mass-spring systems [48,36]. Each mass has connections to the masses next to the adjacent masses, in addition to the usual cascade connection to the adjacent ones. The system can be considered as a linear approximation of a tensegrity beam subjected to tensile and compressive forces. Tensegrities have received significant interest among scientists and engineers in many fields, such as architecture, aerospace, and robotics [30,43], because of their light weight, deployable, shape control, and tunable stiffness properties. The tensegrity beam corresponds to a continuum beam in continuum mechanics, and is considered a fundamental element (or unit) in tensegrity design.

We introduce the double cascade-connected damped mass-spring systems in Section 2, and provide the main outcomes in Section 3. In Section 3.1, we derive the recurrent formula of the system dynamics in the Laplace transform domain, and introduce a similarity transformation to represent the system by the sum of wave motions in Section 3.2, where the properties of the propagation constants are also investigated in detail. The impedance

* Tel.: +81 73 457 8162; fax: +81 73 457 8201.
E-mail address: nagase@sys.wakayama-u.ac.jp

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matching controller is derived from the condition to eliminate reflected waves in Section 3.3. The controller can be constructed from a similarity transformation of the characteristic impedance matrix by a matrix composed of the propagation constants. In Section 4, we discuss the relationship between the system and the tensegrity beam, and a numerical example of vibration control of a tensegrity beam is given in Section 5. Section 6 summarizes and concludes the paper.

In the following, s represents the Laplace operator, and $\mathbb{C}_+$ is the set of complex numbers with positive real part. For notational simplicity, we use the same symbol for a time variable and its Laplace transform ($x(t) \leftrightarrow x(s)$).

2. System configuration

We focus on the wave analysis and control of cascade-connected damped mass-spring systems. Fig. 1 shows a conventional (or single) cascade-connected damped mass-spring system, considered in [48,36]. ℓ represents the number of stages A; and m [Kg], d [Ns/m], and k [N/m] are the mass, damping coefficient, and spring constant, respectively. Each mass (Y) is only connected to the adjacent masses with the damper and the spring (Z). $v_\ell(t)$ [m/s] represents the velocity of the ℓth mass, and x_ℓ [m] represents the displacement ($v_\ell(t) = \dot{x}_\ell(t)$). $f_\ell(t)$ [N] is the reaction force from the right side elements. The reaction force from the left is $-f_{\ell-1}(t)$ in this case.

If we add additional dampers and springs to connect beyond the adjacent masses, the system of Fig. 1 becomes Fig. 2. In this case, beyond the connection with ($\ell-1$)th and ($\ell+1$)th masses, the ℓth mass is connected to the ($\ell-2$)th and ($\ell+2$)th masses through the dampers and springs, Z'. The reaction force from the left is $f'_\ell(t)$ [N]. We assume that the system is uniform, i.e., all the masses, damping coefficients, and spring constants are the same, respectively. The system is a natural extension of the single cascade-connected system to multiple connections. We call this system the double cascade-connected damped mass-spring system, and it can be regarded as a linear approximation of the longitudinal vibration of a cantilevered tensegrity beam. The relationship between the system and the tensegrity beam is given in Section 4.

We investigate the wave analysis and wave control of the double cascade-connected damped mass-spring system shown in Fig. 2.

3. Main results

3.1. Recurrent formula

For lumped parameter systems, the recurrent formula of the dynamics represented in the Laplace transform domain plays a central role in the wave analysis [48,36]. Thus, we first derive a recurrent formula in the Laplace transform domain for the double cascade-connected damped mass-spring system shown in Fig. 2.

To describe the forces acting on the ℓth mass, let the transfer functions $Y(s)$ and $Z(s)$ be

$$Y(s) = ms, \quad Z(s) = \frac{s}{ds+k}. \tag{1}$$

Using $Y(s)$, the inertia force of the ℓth mass is

$$m\dot{v}_\ell(s) = Y(s)v_\ell(s), \tag{2}$$

and using $Z(s)$, the reaction force $f_\ell(s)$ of the mass from the right is

$$\begin{aligned} f_\ell(s) &= \left(k/s+d\right)(v_\ell(s)-v_{\ell+1}(s)) + \left(k/s+d\right)(v_\ell(s)-v_{\ell+2}(s)) \\ &= \frac{1}{Z(s)}(v_\ell(s)-v_{\ell+1}(s)) + \frac{1}{Z(s)}(v_\ell(s)-v_{\ell+2}(s)). \end{aligned} \tag{3}$$

Similarly, the reaction force $f'_\ell(s)$ from the left is

$$f'_\ell(s) = \frac{1}{Z(s)}(v_\ell(s)-v_{\ell-1}(s)) + \frac{1}{Z(s)}(v_\ell(s)-v_{\ell-2}(s)). \tag{4}$$

In the following, we omit the argument s, unless it is explicitly required.

The equation of motion of the ℓth mass is

$$Yv_\ell = -f_\ell - f'_\ell. \tag{5}$$

Setting $\ell \to \ell-2$ in (3), we have the velocity,

$$v_\ell = -Zf_{\ell-2} - v_{\ell-1} + 2v_{\ell-2}. \tag{6}$$

Substituting (4) into (5), and eliminating v_ℓ by (6), we have the reaction force,

$$f_\ell = (-2Y - 3/Z)v_{\ell-2} + (Y + 3/Z)v_{\ell-1} + (2 + ZY)f_{\ell-2}. \tag{7}$$

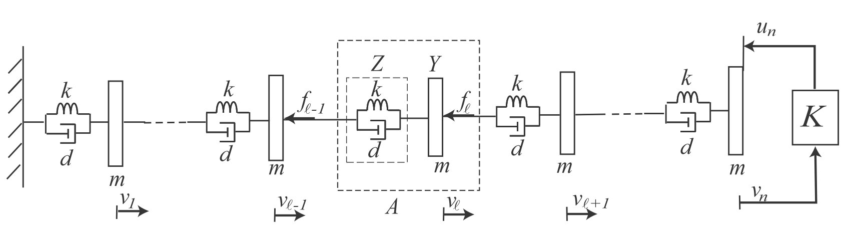

Fig. 1. Single cascade-connected damped mass-spring system.

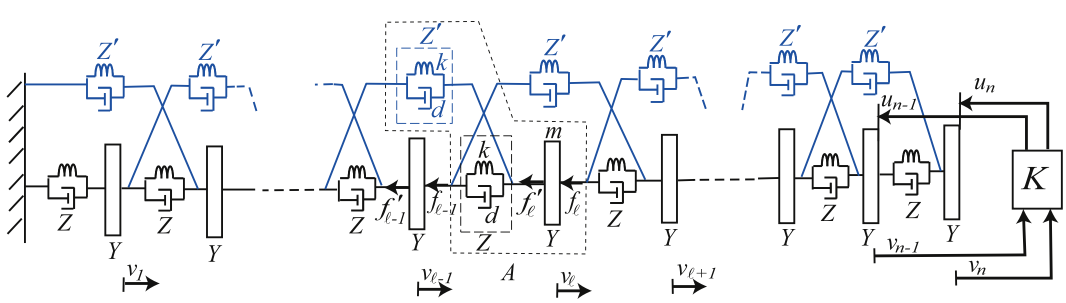

Fig. 2. Double cascade-connected damped mass-spring system.