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An approximation to red blood cells with a model of three-center-combined shells

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ABSTRACT

Red blood cells present a biconcave shape and bear an inner pressure (osmotic pressure) when they are in the static state. In this paper, a model of "three-center-combined shells", which consists of two spherical shells and a toroidal shell, is employed to describe the geometric shape of red blood cells. Surface area and volume of the combined shells model are very close to those measured from experiment. The stress distribution in the cell membrane is formulized as a closed form according to the Novozhilov's theory of the three-center-combined shells. Calculating results in terms of Novozhilov's formula give a good agreement with the numerical results given by ABAQUS when using actual measurements. It is concluded that the combined shells model can well approximate to the biconcave structure of red blood cells. In addition, stress calculation shows that the membrane of biconcave red blood cells can carry bending moments, and the moments reach a maximum value in the vicinity of joint line of the spherical shell and the toroidal shell in the combined shells model.

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1. Introduction

Red blood cells tend to deform easily. They present biconcave shape when at rest and display various different shapes when flow. For example, they become extremely elongated and freely bent when flow into the micro vascular. The average life span of a red blood cell is 120 days. After expansion to sphere, it is ultimately devoid in the spleen [1].

Red blood cells usually bear an inner pressure. The amplitude of the inner pressure can be adjusted by the bidirectional permeability mechanism, and the inner pressure is also referred to as osmotic pressure. During osmosis experiments, when the red blood cells present a biconcave shape, the osmotic pressure is lower and there is less fluid in the cell. When the curvature of red blood cells is positive everywhere (gibbous), the osmotic pressure is higher and there is more fluid in the cell correspondingly.

When the shape of red blood cells is biconcave, there are two areas on the cell surface, which have positive and negative Gaussian curvatures, respectively. The joint of the two areas is called turning line or transition line. Thin shell theory indicates that there exist bending moments in the vicinity of the transition line. Therefore the cell membrane can sustain moments and behaves as a "real membrane" when the shape of red blood cells is biconcave. However, when the cell is spherical, its membrane is incapable of carrying any bending moment, and only provides surface tensions. In this situation it behaves as an "ideal membrane". As an ideal membrane the red blood cells can change their shapes arbitrarily when moving in a blood vessel. The concepts of "real" and "ideal" membrane described above are given by Libai [2].

Thus, it can be inferred that the cell membrane of living body can actively adjust the tensions of its different areas or the stress distribution in it to change itself into or "real" or "ideal" membrane and finally change its shape.

The transformation of the shape of red blood cells has been discussed earlier, for example, in the paper by Beck [3].

A few models have been developed to analyze the mechanical performance of cell membranes. Fung [4] established a variable thickness shell model for red blood cells. Evans [5] proposed a single strain energy function consisting of two terms to simulate the constitutive relation of the membranes of red blood cells. Helfrich [6] proposed a theory of the elasticity of lipid bilayers and explained the biconcave-discoid shape under normal physiological condition. Zarda [7] computed a large elastic deformation of red blood cells on the basis of an assumed model includes the elasticity of the membrane under tensions in its own plane and the bending elasticity. Zhan et al. [8] simulated numerically the instantaneous deformation of red blood cells from biconcave into spherical shape and found a critical osmotic pressure.

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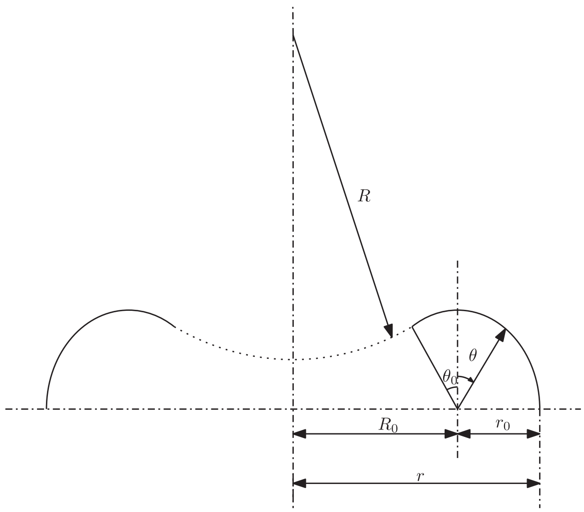

Fig. 1. Schematic diagram of three-center-combined shells.

Therefore, it is important to determine the stress distribution in the membrane of red blood cells when the cells have biconcave shape. However, it is not understood yet how the membrane adjusts its microstructure to form a desired stress distribution. It can be imagined that the adjustment mechanism is very flexible, and allows an arbitrary transformation between the real and the ideal membrane.

It is found in this paper that the biconcave geometry of red blood cells can be approximated in terms of a model of three-center-combined shells. As for the three-center-combined shells, the analytical expressions of stresses and straines have been given by Novozhilov [9] in his monograph. Those expressions are directly employed in this paper to calculate the stress distribution in the membrane of biconcave red blood cells. For better reading, the derivation of those expressions are listed in Appendix A. Some errors in the monograph are modificated. In order to verify those expressions, the ABAQUS software is employed to make a parallel calculation. Measured data and experimental data are used in all the culculations. Within limits of the authors' knowledge, it is not found yet in any literature that the three-center-combined shells are used to model red blood cells.

2. Three-center-combined shells

Three-center-combined shells are an assembly of two spherical shells and a toroidal shell. The shells are symmetrical about their horizontal middle plane. The upper half of the shells is shown in Fig. 1, in which the dotted line represents a spherical shell with radius of R; the solid line represents a toroidal shell, whose radius is r_0 and the rotational radius of its centerline is R_0; the angle at the joint of two kinds of shells is θ_0; the thickness of the shells is h. As can be seen that there are three centers for the half of the combined shells, Novozhilov [9] referred them to as "three-center-combined shells".

3. Red blood cells with biconcave shape

Evans and Fung [10] gave a formula to fit an average cross-sectional shape of red blood cells as follows:

$$\begin{aligned} &\eta = 0.5[1-\xi^2]^{1/2}(C_0 + C_1\xi^2 + C_2\xi^4) \\ &X = 3.91\xi(\mu\mathrm{m}),\ Y = \eta(\mu\mathrm{m}) \end{aligned} \tag{1}$$

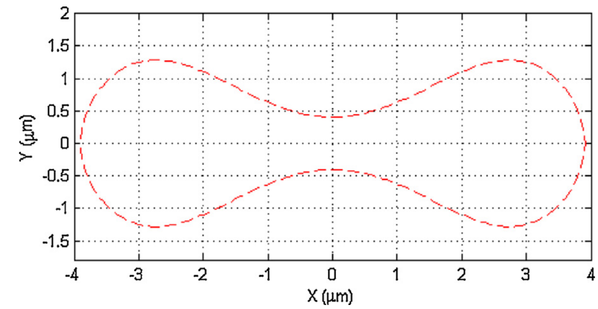

Fig. 2. Red blood cell fitting figure of measured data.

where ξ is in the range of $-1 \le \xi \le 1$, and X and Y are two coordinates; constants $C_0 = 0.207161$, $C_1 = 2.002558$ and $C_2 = -1.122762$, respectively. Above formula is graphed in Fig. 2.

4. Model of three-center-combined shells

By reason of symmetry, a quarter of the cell model is taken to be analyzed and shown in Fig. 3.

The center of spherical shells is located at $(0, Y_0)$ and the center of toroidal shells is at $(X_0, 0)$. According to the formula (1), it is easily determined that $r = 3.91\ \mu$m and $Y_0 = 0.405 + R$. By adjusting the coordinates of the two centers and the radii of the two shells, it is finally determined that the radius of the spherical shells $r_0 = 1.2920\ \mu$m and the associate abscissa is $X_0 = 2.6180\ \mu$m; the radius of the toroidal shells is $R = 3.0150\ \mu$m and the associate ordinate is $Y_0 = 3.4200\ \mu$m; the angle at the joint of the spherical and toroidal shells is $\theta_0 = 0.6533$.

Two graphs depicted by using both Eq. (1) and the combined shells model respectively are illustrated in Fig. 4. In the figure, the red dashed line represents the graphical result of Eq. (1) for measured data, and the blue solid line is the graphical result of the combined shells model. It can be clearly seen from Fig. 4 that the

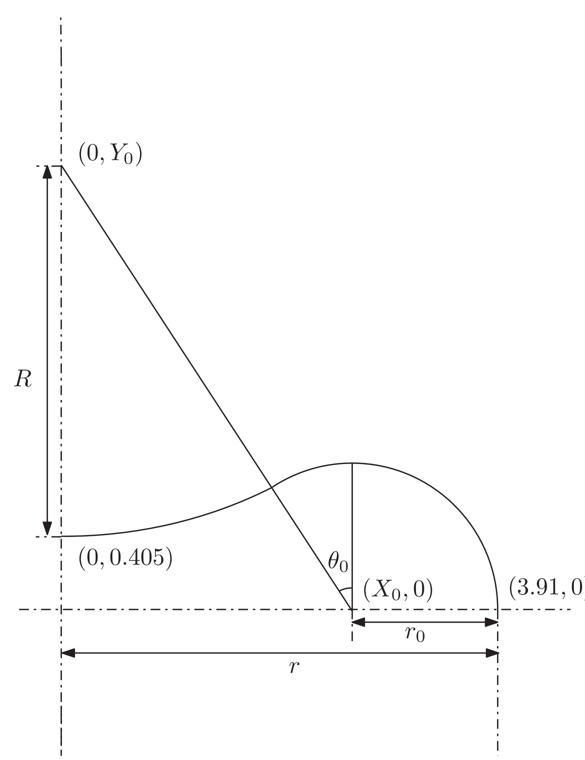

Fig. 3. Calculation model (units of μm).

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