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Crack tip plasticity of a half-infinite Dugdale crack embedded in an infinite space of one-dimensional hexagonal quasicrystal

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ABSTRACT

The present paper is devoted to determining the crack tip plasticity of a half-infinite Dugdale crack embedded in an infinite space of one-dimensional hexagonal quasicrystal. A pair of equal but opposite line loadings is assumed to be exerted on the upper and lower crack lips. By applying the Dugdale hypothesis together with the elastic results for a half-infinite crack, the extent of the plastic zone in the crack front is estimated. The normal stress outside the enlarged crack and crack surface displacements are explicitly presented, via the principle of superposition. The validity of the present solutions is discussed analytically by examining the overall equilibrium of the half-space.

1. Introduction

Quasicrystals (QCs), as a new structure of solid matter, were first discovered by Shechtman et al. [30]. Since then, QCs have become the focus of theoretical and experimental studies in the physics of condensed matter [8]. In the past three decades, a great progress has been made in understanding of the geometric structure and mechanical properties of QCs [6,8]. To date, QCs have been observed to have some desirable properties, such as low friction coefficient [5], low adhesion [25] and high wear resistance [2]. These desirable properties make QCs enjoy a high potential of practical applications to various engineerings. Recently, Kenzari et al. [18] find that QCs can be used as reinforced phases of polymer-matrix composites. Due to wide application prospects of QCs, researches of mechanical properties of QCs become extremely important and necessary.

As a long-standing problem in solid mechanics, crack analysis has been extended to mechanics of QCs in the recent decade [9,11]. Up to now, a lot of research efforts to crack analyses for QCs have been made [11]. For example, with the help of crack theories in conventional linear elasticity fracture mechanics, Li et al. [22] derived an exact analytic solution for a uniformly pressured Griffith crack in decagonal QCs, by means of a general solution. Then, Peng and Fan [26–28], making use of integral transform techniques, investigated the problems of circular cracks in one-dimensional (1D) hexagonal QCs and two-dimensional (2D) decagonal QCs. Later, Fan and Mai [9] gave a comprehensive review of the mathematical theory and methodology of elasticity of QCs and their applications to dislocations and cracks. For the anti-plane crack problems in 1D hexagonal QCs, some works [31,14,15] have been achieved under the framework of elasticity of QCs, to explore the effect of phason field on the deformations of the cracked materials. For the plane problems of an elliptic hole and a crack in three-dimensional (3D) QCs, Gao et al. [12,13] derived the explicit solutions for the phonon-phason coupled fields are obtained in closed forms, by a complex potential approach and the generalized Stroh formalism. Fan et al. [11] made a comprehensive review on the fracture theory of QCs concerning with linear, nonlinear and dynamic crack problems for QCs of various types. Recently, Li [19], using the potential theory method in conjunction with the general solutions, solved the mode I problems of three common planar cracks and presented the elastic fundamental fields as well as some important parameters in crack analysis.

In the context of the classical theory of linear elasticity, the stresses at the crack tip are singular and exceed the yield stress of the material, which is an unrealistic behavior in practice [17,1]. Hence, the real crack problem should be taken into consideration under the framework of plasto-elasticity. To dispose the non-linear crack problem, Dugdale [7] proposed a simple model which considers a strip zone of concentrated plastic deformation in the crack front. In his model, Dugdale assumed that a constant cohesive stress, which is equal to the yield stress of the material, exists in the plastic zone. In fact, Dugdale model is very effective and has been verified by experiments [7,3]. Due to its simplicity and validity, the idea underlying the Dugdale model has been widely

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adopted and developed to dispose a variety of crack or crack-like problems [3,29,33,4,24,16,21]. For instance, applying Dugdale's approach, Maugis [24] successfully explained the transition of JKR-DMT adhesive contact theory. Furthermore, Dugdale crack model is generalized to materials with more complicated yield surfaces, where the Tresca or von Mises yield criterion is required to satisfy in the crack tip plastic zone [3,16]. In recent years, some efforts on the Dugdale crack of QCs have been made by Fan and his coauthors [10,23,32], to investigate the size of plastic zone. To the best of authors' knowledge, however, the problem of the half-infinite Dugdale crack embedded in an infinite space of 1D hexagonal QC has not been studied.

The purpose of this paper is to study a half-infinite Dugdale crack in 1D hexagonal QC. The crack is assumed to be subjected to a pair of equal but opposite line phonon forces in the upper and lower crack surfaces. The Dugdale hypothesis in company with the elastic results for a half-infinite crack is applied to evaluate the extent of plasticity. With the aid of superposition principle, the normal stress outside the enlarged crack and crack surface displacements (CSDs) are explicitly presented. Numerical calculations are performed to examine the validity of the present solutions and to show the influence of some parameters on the distributions of some important physical quantities. Furthermore, the present solutions can be used to serve as benchmarks for computational fracture mechanics.

2. Preliminaries for a half-infinite crack

Consider an infinite space of 1D hexagonal QC weakened by a half-infinite plane crack in parallel with the isotropic plane. In the Cartesian coordinate system (x, y, z), the atoms of 1D QCs are arranged periodically in the planes parallel to the xoy plane and quasiperiodically in the z-direction. For convenience, the cracked plane is assumed to be coincident with the plane $z=0$ (denoted by I hereafter). For simplicity, the region of the crack is symbolized by

$$S \equiv \left\{(x, y, z) \mid -\infty < x < +\infty,\ y \geq 0,\ z = 0\right\}. \tag{1}$$

On the upper and lower crack lips are applied two pairs of equal but opposite line loads $\pm P_1^0$ (phonon) and $\pm P_2^0$ (phason) respectively along the lines $y=y_m$ ($m=1, 2$; $y_m>0$, $-\infty<x<+\infty$, $z=0$), as illustrated in Fig. 1. Therefore, the half-infinite crack problem is reduced to a plane strain problem and all the physical quantities would be independent of the variable x.

In view of the symmetry with respect to the plane $z=0$, the problem in question can be converted into a mixed boundary value problem (MBVP) of the half-space $z \geq 0$, with the following boundary conditions prescribed on the plane $z=0$:

$$\left.\begin{array}{ll} \forall (x, y) \in S: & \sigma_{zm} = P_m^0 \delta(y - y_m); \\ \forall (x, y) \in I - S: & u_{zm} = 0; \\ \forall (x, y) \in I: & \tau_z = \sigma_{zx} + i\sigma_{zy} = 0,\ i = \sqrt{-1}, \end{array}\right\} \tag{2}$$

where $\sigma_{z1}(\sigma_{z2})$ and $u_{z1}(u_{z2})$ respectively denote the normal stress and the displacement in the z-direction in phonon (phason) field; τ_z is the complex phonon shear stress. Hereafter, the indices 1 and 2 indicate the physical quantities associated with the phonon and phason fields, respectively, unless otherwise stated.

This MBVP has been investigated by [19], as a direct application of the fundamental solutions. For better usage, the normal stress on the crack plane, stress intensity factors (SIFs) and CSDs are recalled here. The generalized stress components on the intact region $I-S$ read

$$\sigma_{zm}|_{z=0} = -\frac{P_m^0}{\pi(y_m - y)}\sqrt{\frac{y_m}{-y}},\ (y < 0). \tag{3}$$

Defining the SIFs of mode I crack problem as

$$k_m = \lim_{y \to 0^-} \sqrt{-y}\sigma_{zm}(x, y, 0),$$

we arrive at

$$k_m = -\frac{P_m^0}{\pi\sqrt{y_m}}. \tag{4}$$

The CSDs in this case are of the form

$$u_{z1} = \frac{P_1^0 g_{22}}{\pi^2 A} I(y; y_1) - \frac{P_2^0 g_{12}}{\pi^2 A} I(y; y_2) \tag{5a}$$

and

$$u_{z2} = -\frac{P_1^0 g_{21}}{\pi^2 A} I(y; y_1) + \frac{P_2^0 g_{11}}{\pi^2 A} I(y; y_2), \tag{5b}$$

where $I(y; y_m)$ is defined by

$$I(y; y_m) = \ln\frac{\sqrt{y + y_m}}{|\sqrt{y} - \sqrt{y_m}|}\ (y > 0).$$

It is noted that the constants A and g_{ij} ($i=1, 2$; $j=1, 2$) in (5) are material parameters which are defined by [19] and presented in Appendix A. The constitutive laws of 1D hexagonal QC along with some auxiliary parameters are presented in Appendix A as well.

3. Half-infinite Dugdale crack

Consider the aforementioned crack only subjected to a pair of equal but opposite phonon loads $\pm p_0$ exerted on the line $y=y_1$, namely,

$$p_1^0(x, y) = -p_0\delta(y - y_1),\ p_2^0(x, y) = 0. \tag{6}$$

It is necessary to note that phason loads are not considered in this case, on account of the weak effect of phason field on the deformations of the cracked material [20,19]. The same phason actions often appear as about 1% perturbations of the standard stress field. Therefore, phason loads are difficult to be controlled from the exterior, at the macroscopic scale [19]. In fact, the analytical approach for the case of phason loads can be readily extended from the present work, without any difficulty.

Owing to the external source, plastic deformations occur at the crack front since the stresses in the neighborhood of the crack tip exceed the yield stress of the material constituting the body.

Since the problem in question is a plane strain one, the crack front after deformation would be parallel to the original one. In

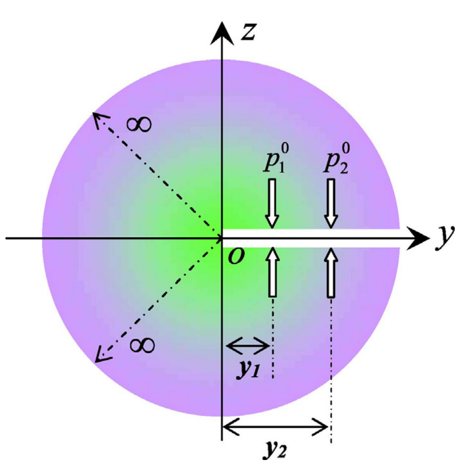

Fig. 1. A schematic figure for a half-infinite crack subjected two pairs of line loads $\pm P_1^0$ (phonon) and $\pm P_2^0$ (phason) applied along the lines $y=y_1$ and $y=y_2$, respectively.