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Deformation of thin straight pipes under concentrated forces or prescribed edge displacements

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ABSTRACT

The purpose of this paper is to present an efficient analytic method for obtaining the deformation of thin straight pipes, subjected to prescribed edge displacements or concentrated loads.

The approach uses the mixed formulation where unknown functions are combined with trigonometric terms. A variational procedure is used to obtain the system of ordinary differential equations. For the applied load a Fourier approach is used to represent the load as an analytical function. For the prescribed displacement, three solutions for the ovalization are evaluated and a method based on energy contribution of each term is used to obtain their superposition.

In contrast to finite element method the proposed method is efficient and can be applied to other boundary condition problems leading to continuous displacement and stress fields with a low number of unknowns. Comparisons with experimental and finite element procedures show good agreement that enhances the merits of the analytical solutions proposed.

The value of this method is based on solving the differential equations rather than using commercial codes. So far, the solution of prescribed edge displacements has been limited to one term. This paper discusses how to add further terms using the mixed formulation, thus, presenting a novel procedure.

1. Introduction

Stress analysis using simple formulation solutions is an important area of research in modeling tools for structural engineering. Applications to pipe engineering are of particular motivation given its simple geometry albeit combined with complex loads, demanding accurate formulations dealing with the singularity of some external force systems. This is the case of the effect of radial loads on cylindrical shells, their study being important to the engineering design and installation procedures for cylindrical pressure vessels and pipes. Evaluation methods for the structural effect of such loads are essentially numerical and experimental. The former of these deals mainly with finite element techniques. Works of Oñate [1], Bathe and Almeida [2], Mackenzie et al. [3], Fonseca et al. [4,5], Nguyen-Thanh et al. [6], Qian et al. [7] are important contributions to the research in this area. Application of commercial codes also results in an important amount of publications.

Any numerically based methodology that is applied to structural analysis and design needs to be certified for accuracy and field of validation. Experimental techniques are useful for this objective, as it is possible to achieve realistic results, provided that the load conditions of the project can be reproduced as close as possible to the expected force system in the design problem. An extended experimental analysis program using strain gauge techniques for pipe bends was implemented by Hose and Kitching [8]. Furthermore, Kitching and Hughes [9] studied the elastic behavior of a cylindrical shell subjected to a radial load. Rhodes [10] described an extensive list of contributions from researchers in the University of Strathclyde, where the stress state in cylindrical shells using experimental techniques has been investigated for several years.

The laser optical technique is also an experimental procedure that allows displacement field assessment with a noncontact procedure, using image processing methods to read the displacement and respective gradients from a fringe pattern generated with laser illumination of the specimen [11]. Recently, high accuracy and wide temperature measurements by non-contact speckle extensometer with a resolution of 1 μm were attained by comparison with a linear laser encoder by Kobayashi and Yamaguchi [12]. These techniques are used to validate and compare with numerical methods as shown in Section 3.

The search for exact solutions in pipe engineering for cylindrical shells, solving analytically the differential equations for structural

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Nomenclature

$a_i(x)$	ovalization function
$b_i(x)$	warping function
K	elastic pipe properties matrix
E	Young's modulus
F	internal force vector
M_θ	hoop bending moment
N_x	longitudinal membrane force
$N_{x\theta}$	membrane shear force
t	pipe thickness
r	pipe transverse section radius
R	coefficients matrix
U	total internal energy
u	displacement field
u	longitudinal shell displacement
v	circumferential shell displacement
w	transverse shell displacement
$\boldsymbol{\varepsilon}$	deformation vector
$\boldsymbol{\nu}$	poisson's ratio
$\boldsymbol{\sigma}$	vector of internal stresses
$\boldsymbol{\Omega}$	the integral domain

analysis in thermal and mechanical problems, has also been published by several authors. Donnel [13] developed an eight-order differential equation for the determination of the critical strength of cylinders with simply supported edges under torsion. Batdorf [14] presented a simplified method of elastic stability analysis for thin cylindrical shells by solving Donnell's equation of equilibrium for various types of loading and boundary conditions. Millard and Roche [15] presented an irreducible method that involved optimized analytical functions for propagation of ovalization along a straight pipe. Jabbari et al. [16] published analytical solutions for mechanical and thermal stresses using complex Fourier integral and Bessel functions. And recently Nasrekani et al. [17] presented the problem of elastic buckling stress of cylindrical shells under axial loads solving a set of differential equations using the perturbation method.

In this study a procedure combining unknown functions with Fourier trigonometric expansions is presented, where a system of differential equations is analytically solved. The equilibrium equations are derived by the virtual work principle and then the stability equations are obtained. The method is based on a mixed definition where a lower number of unknowns are used to reach the solution when compared to the finite element techniques [18,19].

2. Mixed formulation solution for deformation analysis of thin pipes

2.1. The mixed formulation

In the mixed formulation the vector of unknowns combines stresses and displacements of the deformed structure where a constitutive equation relates the internal force field to the corresponding deformations. This is expressed by Eq. (1):

$$\{\boldsymbol{F}\} = [\mathrm{K}]\{\boldsymbol{\varepsilon}\} \tag{1}$$

where $\{\boldsymbol{F}\}$, [K], and $\{\boldsymbol{\varepsilon}\}$ are, respectively, the internal force vector, the elasticity matrix, and the deformation vector. This last vector can be expressed as $\{\boldsymbol{\varepsilon}\} = L\{\boldsymbol{u}\}$, involving the displacement vector $\{\boldsymbol{u}\}$ and a differential operator L, containing differential expressions

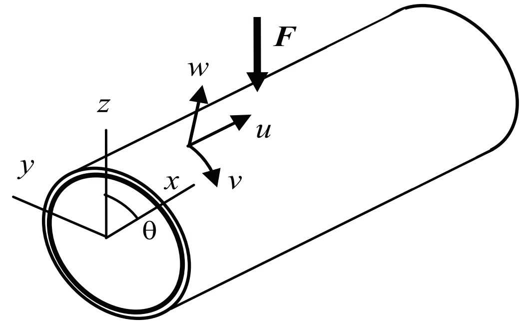

Fig. 1. Shell displacements.

for the evaluation of deformations. The previous relation can be written as:

$$[\mathrm{K}]^{-1}\{\boldsymbol{F}\} = L\{\boldsymbol{u}\} \tag{2}$$

If Eq. (2) is pre-multiplied by the transposed force vector $\{\boldsymbol{F}\}^{\mathrm{T}}$, a new formulation involving energy terms is obtained:

$$\{\boldsymbol{F}\}^T[\mathrm{K}]^{-1}\{\boldsymbol{F}\} = \{\boldsymbol{F}\}^T L\{\boldsymbol{u}\} \tag{3}$$

The total energy U stored in the deformed solid is obtained by calculating the integral expression over the volume of the solid:

$$U = \int_{Vol} \{\boldsymbol{F}\}^T[\mathrm{K}]^{-1}\{\boldsymbol{F}\} - \{\boldsymbol{F}\}^T L\{\boldsymbol{u}\}\, dV \tag{4}$$

Some assumptions that need to be considered in order to evaluate the solution are:

- That the shell is thin (what is normal to the shell surface before deformation remains so after).
- That the pipe is inextensible in the θ-direction (circumferential).

The distortion of the pipe is characterized by the following shell displacements as shown in Fig. 1:

The displacement field is defined by u as the longitudinal displacement, v the circumferential shell displacement and w the transverse shell displacement. The functions considered are:

$$\begin{aligned} u(x,\theta) &= \sum_{i=2}^{n} b_i(x)\cos(i\theta) \\ v(x,\theta) &= \sum_{i=2}^{n} -\frac{a_i(x)}{i}\sin(i\theta) \\ w(x,\theta) &= \sum_{i=2}^{n} a_i(x)\cos(i\theta) \end{aligned} \tag{5}$$

In the previous expansions the unknown coefficients $a_i(x)$, $b_i(x)$ are the amplitudes for the ovalization and warping displacements. These terms are included in the Fourier θ-expansions, but they are dependent only on x.

2.2. The variational solution obtained by total energy minimization

Considering N_x as the longitudinal membrane force, $N_{x\theta}$ as the membrane shear force, M_θ as the hoop bending moment, the