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Modeling gravitational collapse of rectangular granular piles in air and water

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ABSTRACT

The present paper is concerned with the two-dimensional collapse of piles of granular materials, a problem analogous to the classical dam break problem in hydraulics. This study is intended to aid in the development of constitutive equations and modeling procedures that can be applied to predict various flows involving high concentration liquid–particle mixtures. We consider the granular collapse as a test problem and attempt to validate our modeling by comparing our predictions with previously published granular collapse experiments. The time-dependent evolution of the collapsing granular piles is calculated by making use of COMSOL, a commercial finite element code that is designed to handle a wide variety of Multiphysics problems. We begin by considering the collapse of a rectangular block of dry granular material and calculate the temporal evolution of the free surface by making use of the Level Set method. Good agreement is found between these predictions and the laboratory experiments of Balmforth and Kerswell (2005). The collapse of granular material submerged in a water is then investigated using a Mixture Model approach. The experiments of Rondon et al. (2011) revealed drastically different collapse periods depending upon whether the initial pile was in a loose or a dense, compacted state. The simple Mixture Model approach gave reasonably good predictions of the Rondon et al. (2011) experiments for the case of initially loose piles that collapsed in about a second, but it was unsuccessful in simulating the collapse of the initially dense piles that were observed by Rondon et al. (2011) to take around 30–40 s. Some simple empirical modifications to the material constitutive behavior were able to roughly predict such long collapse times, but a more comprehensive and detailed investigation of the phenomenon is warranted.

1. Introduction

The present paper describes analyses of the collapse of piles of granular material. This problem can be regarded as analogous to the classical dam break problem in hydraulics. In a geophysical context it could be associated with a highly idealized model of the collapse of a vertical cliff.

There have been several previous studies of the collapse of piles of granular material onto horizontal surfaces. Lube et al. (2004) considered granular material initially contained in a vertical circular cylindrical tube that was rapidly lifted to release the grains and allow them to spread on to a horizontal surface. One intended goal was to better understand various features of pyroclastic flows and debris avalanches. Several different granular materials, including sand, salt, couscous, rice and sugar were used in their studies. The experiments varied the initial aspect ratio (column height to radius) of the pile and examined the subsequent evolution of the spreading pile. Scaling laws for the final runout were devised.

Lajeunesse et al. (2004) carried out laboratory experiments similar to those of Lube et al. (2004). Spherical glass beads of various sizes were used as the granular material. The effect of basal roughness on the spreading of the pile was studied. Again, the effect of the initial column aspect ratio on the evolution of the collapse was examined.

Balmforth and Kerswell (2005) performed extensive investigations of the collapse of rectangular piles of granular materials confined between two parallel walls. The flow was initiated by withdrawing a swinging gate or a sliding door, and the granular pile slumped and spread on a horizontal floor. The granular materials tested comprised glass beads, polystyrene balls and irregular shaped grit. A simple two-dimensional depth averaged model based on a basal friction coefficient was able to capture several aspects of the experimental flows.

A number of studies have been carried out to calculate the collapse of granular columns using discrete particle numerical approaches. Zenit (2005) made use of a two-dimensional discrete

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element approach to consider the flow of inelastic, frictional circular disks. His numerical results qualitatively reproduced the previous experimental results of Lube et al. (2004) and Lajeunesse et al. (2004).

Staron and Hinch (2005) investigated two-dimensional granular column collapse and spreading using the Contact Dynamics approach in which individual grains interacted through hard-core repulsion and Coulombic friction. They studied the effects of the column aspect ratio and basal friction. The diameters of the grains were uniformly distributed in a small interval such that the ratio of the minimum to maximum diameter $d_{min}/d_{max} = 2/3$. The results of their calculations were in good agreement with the experiments of Lube et al. (2004) and Balmforth and Kerswell (2005). In a later paper, Staron and Hinch (2007) continued their two-dimensional simulations to further examine the effects of initial conditions and the details of the interactions between grains.

Lacaze and Kerswell (2009) have used the commercially available discrete element software package EDEM to model the axisymmetric collapse of cylindrical columns of uniform, spherical, inelastic, frictional particles. Particles were released over a horizontal plane that was roughened by a monolayer of spherical particles fixed to the base. Stress and strain-rate tensors were extracted from the collapsing flow data and Lacaze and Kerswell (2009) used this information to make comparisons with the rheological model of Jop et al. (2006).

Larrieu et al. (2006) attempted to analyze the collapse of columns of grains by dividing the flow into two parts. They applied depth-averaged equations to the thin, horizontally flowing layer of material that was subjected to Coulomb friction at the bed. They added material to the flow during the time corresponding to the free fall of the column. The analysis was able to predict the final shape of the deposit and the correct dependence of the final runout upon the initial aspect ratio H/L, where H and L are the initial height and length of the column.

Lagree et al. (2011) studied the two-dimensional collapse of dry granular materials by casting the governing equations in the form of the Navier–Stokes equations and examining the effects of various rheological models. They considered rheologies due to Bagnold, Bingham, a constant friction model and the $\mu(I)$ model of Jop et al. (2006). The $\mu(I)$ model gave the best agreement with experiments and discrete element numerical computations.

Meruane et al. (2010) studied the effects of an interstitial fluid during the collapse of rectangular piles of granular material submerged in fluids. They considered a two-phase mixture theory model and made comparisons with laboratory experiments involving both air and water as the interstitial fluid.

Rondon et al. (2011) carried out experiments to investigate the effect of interstitial liquids on the collapse of granular columns. In particular, they found that varying the initial solids concentration could change the temporal evolution of the pile collapse process drastically. In their experiments for cases in which the particles were initially loose, the collapse was over within the order of a second or two, whereas for the dense case, the collapse could take as long as 30–40 s. To provide some perspective for the time scales involved in the experiments of Rondon et al. (2011), we note that their experiments were 'bench-top scale' and involved granular piles that were several centimeters in height.

1.1. Outline of present paper

The end goal of the present work is the development of appropriate constitutive relations and modeling approaches that can be used to predict the flows of mixtures of high concentration solid particles and fluids. A number of constitutive assumptions and modeling procedures are proposed in the present paper. We consider the particular example of the collapse of initially rectangular piles of granular material and use it as a test problem in an attempt to validate our modeling procedures by making comparisons with previously published granular collapse experiments. The time-dependent evolution of the collapsing granular piles is calculated by making use of COMSOL, a commercial finite element code that is designed to handle a wide variety of Multiphysics problems. We begin by considering the simplest case of the collapse of a rectangular block of dry granular materials. The mass and momentum conservation equations are set down in combination with an assumption for the form of the stress tensor. We determine the temporal evolution of the free surface of the dry granular material by making use of the Level Set method. The results of the computations are compared with laboratory experimental measurements. With the results of the dry collapses as a background, we then move on to study the collapse of granular material submerged in a liquid by using a Mixture Model approach. The case of an initially loose pile is calculated and compared with the experiments of Rondon et al. (2011). It was mentioned earlier that Rondon et al. (2011) found that the period over which the collapse transpired was strongly affected by the initial solids concentration of the submerged pile. We investigate simple empirical modifications to the material constitutive behavior that can produce such effects.

2. Modeling of dry granular collapses

As a precursor to the presentation of the more complicated problem of modeling collapses of piles of mixtures of solid particles and liquids we describe a simple approach to deal with the case of dry granular materials.

2.1. Governing equations

We begin by writing down the governing equations. The numerical solutions will involve a rectangular flow domain that is divided into two regions, one consisting of pure fluid, and the other comprised a mixture of densely packed particles and interstitial fluid.

2.1.1. Mass and momentum conservation equations

In general, we can express the conservation of mass equation as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \tag{1}$$

and the conservation of linear momentum is usually expressed as

$$\rho \left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} \right] = -\nabla p + \nabla \cdot \left[\mu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T) - \left(\frac{2}{3}\mu - \kappa_{dv} \right) (\nabla \cdot \boldsymbol{u})\boldsymbol{I} \right] + \boldsymbol{F} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \boldsymbol{F} = \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{F}, \tag{2}$$

where ρ is the mass density, $\boldsymbol{u}$ is the velocity, t is time, p is pressure, $\boldsymbol{I}$ is the unit tensor, the superscript T denotes the transpose, $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\tau}$ is the viscous part of the stress tensor, $\boldsymbol{F}$ is the body force vector, μ is the dynamic shear viscosity, and κ_{dv} is termed the bulk viscosity. In the present work we take κ_{dv} to be zero.

The stress tensor $\boldsymbol{\sigma}$ can be written in tensor notation as

$$\sigma_{ij} = -p\delta_{ij} + 2\mu\dot{\epsilon}_{ij} - \left[\frac{2}{3}\mu - \kappa_{dv} \right] \dot{\epsilon}_{kk}\delta_{ij}, \tag{3}$$

where the strain-rate tensor is defined as

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right], \tag{4}$$

and u_i is the velocity component in the x_i-direction.