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An invariant based damage model of stress-softening

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ABSTRACT

A phenomenological model to predict the Mullins stress-softening effect in an isotropic, incompressible, hyperelastic rubber-like material is proposed which describes deformation induced microstructural damage and the same is characterised by a simple exponential softening function. The proposed isotropic damage function depends on the maximum previous value of the first invariant of the left Cauchy–Green deformation tensor. The proposed model of softening is illustrated with the theory of Gent material model and finally it is validated with experimental data provided in the literature. The model shows a simple functional form and brings out the interrelation between other models of this type.

1. Introduction

Natural and synthetic elastomers exhibit various inelastic behaviours such as stress relaxation, creep, hysteresis and stress-softening, i.e. Mullins effect. The stress required during the primary loading is always greater than that required for subsequent reloading for the same amount of deformation. This stress-softening phenomenon is known as Mullins effect which was first recorded by Bouasse and Carriére (1903) and then extensively studied by Mullins (1947).

The Mullins stress-softening effect is considered to be due to a damage mechanism in a rubber-like material undergoing deformation for the first time from its virgin state and the softening of the material occurs progressively with the deformation. Mullins and Tobin (1957) first developed a quantitative phenomenology of the two-phase model namely hard phase and the soft phase. In the process of deformation the equivalent amount of hard phase converts into soft phase. The extent of transformation depends only on the extent of maximum deformation it has undergone in the strain history in its selective memory. The deformation beyond any maximum previous deformation updates the material memory to its current value. This selective memory property is a continuous function of maximum previous ever deformation. However, they did not provide a direct physical interpretation for their model. Johnson and Beatty (1993) also followed the same formulation of hard and soft phase and provided justification of the phase transformation.

A micromechanical scheme of polymeric network reinforced with fine particles, idealised as rigid, and connected by two different types of chains namely elastic and breakable, was proposed in Tommasi et al. (2006). Probabilistic network alteration of the two types of chains was proposed considering the parent model as a neo-Hookean material. Their model was further improvised for healing and hysteresis (D'Ambrosio et al., 2008). Marckmann et al. (2002) proposed a network alteration theory by modifying the molecular network model presented in Arruda and Boyce (1993). First invariant of the left Cauchy–Green deformation tensor is used to describe the network alteration (Tommasi et al., 2006; D'Ambrosio et al., 2008; Marckmann et al., 2002). Zúñiga and Beatty (2002) proposed a strain intensity based model which sets selective memory by the maximum previous strain measure and later Zúñiga (2005) proposed energy based phenomenological model for stress-softening and established an interrelation with Ogden and Roxburgh (1999) pseudo elastic model. Diani et al. (2006) proposed a model for anisotropic damage by combining the network alteration theory of Marckmann et al. (2002) with a constitutive law that uses material directions. Haughton and Merodio (2009) examined the influence of localised strain softening (LSS) on the bifurcation of inflated thin-walled cylinders under axial loading with particular reference to the mechanical response of arteries that are weakened by Marfan's syndrome. They investigated the behaviour characteristics for a specific LSS material model with particular emphasis on the effect of local strain softening compared with a rubber-like material modelled with a neo-Hookean strain energy function.

In order to apply the softening effect, both in boundary value and dynamical problems, a simpler yet predictive model of stress-softening is needed. In the present study, a simplified, three-dimensional, first invariant of left Cauchy–Green deformation

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tensor based damage model for stress-softening for an isotropic, incompressible rubber-like material is proposed. The proposed model is used with the Gent (1996) parent material model. The proposed material model is compared and validated with several existing static and dynamic experimental results.

This paper is organised as follows. Section 2 describes mathematical preliminaries for the constitutive modelling of stress-softened, isotropic material undergoing homogeneous deformation and in the same section an invariant based stress-softening function is proposed for finite deformation. In Section 3, a constitutive theory for a general class of incompressible, isotropic, limited elastic material is presented. Here we restrict our present study to the special class of material for which the deformation is bounded by the first invariant of the deformation tensor. Section 4 validates the proposed model with some quasistatic homogeneous deformation data and then we extended validation of the model for some dynamical experimental data. Some concluding remarks are given in Section 5.

2. Proposed softening model

An incompressible, isotropic, hyperelastic, rubber-like material undergoing an isothermal process of deformation, is characterised by a scalar strain energy density function $W = W(I_1, I_2)$ from which the constitutive relations may be derived. Here I_1 and I_2 are the first and second invariants of the left Cauchy–Green deformation tensor $\mathbf{B}$, respectively, with $\mathbf{B} = \mathbf{F}\mathbf{F}^T$ and $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X}$ is the deformation gradient tensor.

The Cauchy stress $\mathbf{T}$ is given by

$$\mathbf{T} = -p\mathbf{I} + \beta_1\mathbf{B} + \beta_{-1}\mathbf{B}^{-1} \tag{1}$$

where p is unknown hydrostatic pressure, $\mathbf{I}$ is identity tensor and the material parameters are defined as

$$\beta_1 = 2\left(\frac{\partial W}{\partial I_1}\right), \quad \beta_{-1} = -2\left(\frac{\partial W}{\partial I_2}\right) \tag{2}$$

Invariants can also be expressed in terms of the principal stretches λ_1, λ_2 and λ_3, as

$$\begin{aligned} I_1 &= tr \quad \mathbf{B} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \quad I_2 = \frac{1}{2}(I_1^2 - tr \quad \mathbf{B}^2) \\ &= \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2, \quad I_3 = \det \mathbf{B} = \lambda_1^2\lambda_2^2\lambda_3^2 = 1 \end{aligned} \tag{3}$$

The stress-softening rubber-like material keeps the maximum previous ever deformation in its memory and it keeps on changing it if the deformation exceeds its previous value. The strain intensity based models (Zúñiga and Beatty, 2002; Beatty and Krishnaswamy, 2000) set this selective memory by the maximum previous ever strain measure at the material point $\mathbf{X}$. Motivated with this concept of changing memory with the deformation, a simple and effective model is proposed, where the selective memory depends only on the maximum value of the first invariant I_1 of the left Cauchy–Green deformation tensor. The selective memory function is defined as $\theta = \theta(I_1)$. At a material point $\mathbf{X}$ the maximum previous ever deformation is defined by $\theta_{\max} = \Theta = \max_{0\le s\le t}\theta(t)$, where the material is subjected to a deformation history up to the current time t, and s is a running time variable. Consequently, the damage may be quantified by a function which takes care of both current deformation θ and maximum previous ever deformation $\Theta = \theta(I_{1M})$, where I_{1M} is the maximum value of I_1 at the maximum preconditioned deformation. The response for stress-softened material for an elastic deformation for which $\theta < \Theta$ is defined by a scalar valued softening function $F\left(\theta;\Theta\right)$ as

$$\boldsymbol{\tau} = F(\theta;\Theta)\mathbf{T} \tag{4}$$

where $\boldsymbol{\tau}$ is the Cauchy stress in the stress-softened material and $\mathbf{T}$ is as defined in Eq. (1) for the virgin response. The softening function $F(\theta;\Theta)$ exhibits a monotone increasing nature with the deformation. Deformations for which $\theta < \Theta$, the softening function satisfies the condition

$$0 < F(\theta;\Theta) < 1; \quad F(\Theta;\Theta) = 1 \tag{5}$$

This softening function characterises the change in microstructure due to damage, and it begins immediately with deformation from virgin state of the material. From Eqs. (4) and (5), it is concluded that the stress responses are same (i.e. $\mathbf{T} = \boldsymbol{\tau} = 0$) at the undistorted state and at each softening point where $\theta = \Theta$. But in course of loading from virgin undistorted state, the current measure of deformation θ is same as that of maximum previous ever deformation Θ, i.e. $\theta = \Theta$ and Eq. (4) reduces identically to Eq. (1). This observation is valid for any current deformation beyond the maximum previous ever deformation for the first time, i.e. $\theta > \Theta$ and subsequently, the current deformation is the maximum previous deformation. Notice that from Eqs. (4) and (5), ratios of the nontrivial physical components of stress-softened material and that of the virgin (or parent) material may be obtained as

$$\frac{\tau_{ij}}{T_{ij}} = F(\theta;\Theta) \le 1, \quad i,j = 1,2,3, \quad \text{no sum,} \tag{6}$$

the equality holding only for $\theta = \Theta$. Thus, it is clear that the stress-softened response is always less than that of the virgin material for the same amount of deformation. This fact is found experimentally and will be described with a few examples.

Thus far, the general property of the softening function is discussed and the physical interpretation of the function to characterise the damage is provided. Now, an exponential softening function is defined as

$$F(\theta;\Theta) = e^{-b\sqrt{\Theta-\theta}} \tag{7}$$

where b is a positive material constant called the softening parameter and choosing the simplest functional form of the current deformation measure given by

$$\theta(I_1) = I_1 \tag{8}$$

the first invariant of left Cauchy–Green deformation tensor. It is phenomenologically postulated that the microstructural changes in the material coordinate is taken care by the softening function. Thus, the maximum previous ever deformation is $\Theta = \max_{0\le s\le t} I_1(s) = I_{1M}$ and in view of this, the softening function defined in Eq. (7) reduces to

$$F(\theta;\Theta) = e^{-b\sqrt{I_{1M}-I_1}} \tag{9}$$

Thus, for any deformation for which $3 \le I_1 \le I_{1M}$ the stress response of a softened material is governed by Eqs. (1), (4) and (9). Next, Gent (1996) material model is used to study the stress-softening behaviour due to its simplicity and applicability to wide range of limited elastic materials.

3. The stress-softened Gent material model

The constitutive equation for general class of incompressible, isotropic, homogeneous, elastic material is defined in Eq. (1). To incorporate limited extensibility in the equation, the response functions are defined as $\beta_1(I_1, I_2; I_m) = \mu(I_1; I_m)$, $\beta_{-1}(I_1, I_2; I_m) = 0$ where I_m is the limiting extensibility constant that bounds the first invariant so that $I_1 < I_m$ for all deformations of the material. Thus, the constitutive equation for special class of materials of limited extensibility simplifies to

$$\mathbf{T} = -p\mathbf{1} + \mu(I_1; I_m)\mathbf{B} \tag{10}$$