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Vibration analysis of curved graphene ribbons based on an elastic shell model

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ABSTRACT

An orthotropic elastic shell model is developed to study the vibration characteristics of curved graphene ribbons (CGRs). The effect of a small length scale is incorporated in the formulations using the gradient elasticity theory. Novozhilov's linear shallow shell theory is used and it is assumed that CGR is simply supported. Analytical solution to the equations is proposed to obtain the frequencies of CGRs. The vibrational properties of CGRs are investigated with respect to the variations of various parameters. Results indicate significant dependence of natural frequencies on the curvature change as well as the modes being considered.

1. Introduction

The carbon-based nanostructures such as fullerene (Kroto et al., 1985), carbon nanotube (Iijima, 1991) and graphene (Novoselov et al., 2004) have been received enormous attention in recent years due to the wide diversity of their structural forms and unusual properties. The varieties of carbon nanostructures such as spheres, cylindrical tubes and planes have different behavior in relation to the dimensionality of the system. Recently, a new type of carbon-based nanostructures which is called curved graphene ribbon (CGR) has been studied by different researchers (Kholmanov et al., 2009; Kato et al., 2010; Gosálbez-Martínez et al., 2011; Belonenko et al., 2011). CGR is an intermediate structure between a carbon nanotube and a flat graphene sheet (Fig. 1). Recently, the fabrication of curved graphene ribbons has been reported by unzipping carbon nanotubes (Kosynkin et al., 2009; Jiao et al., 2009). This opens the way toward the study of the effect of curvature on the behaviors of graphene ribbons.

Right now, the CGR's electronic and magnetic properties have been widely studied. For example, the electronic properties of electrons in flat and curved zigzag graphene nanoribbons have been investigated using a tight-binding model within the Slater Koster approximation, including spin–orbit interaction (Gosálbez-Martínez et al., 2011). In another work, Belonenko et al. (2011) studied the electronic spectrum and tunneling current in curved graphene nanoribbons. However, the CGR's vibrational properties and the Raman spectra are not studied at the present time although they are important.

For mechanical modeling of the nanostructures, the classical (local) continuum models are deemed to fail, because these models only contain bulk material properties and the material properties related to microstructures are neglected. Hence, these local theories are unable to depict the influence of nanoscale effects when the size of a body enters into the micro- or nano-range. To deal with size-dependent material properties, the classical elasticity theory has been extended from various viewpoints. Different non-classical theories such as couple stress theory (Mindlin and Tiersten, 1962; Toupin, 1964), Cosserat continuum (Yoshiyuki, 1968), nonlocal elasticity (Eringen, 1983) and gradient elasticity (Aifantis, 2009) have been developed. The gradient elasticity theory provides extensions of the classical equations of elasticity with additional higher-order spatial derivatives of strains, stresses and/or accelerations (Askes and Aifantis, 2011). It seems that the gradient elasticity theory could potentially play a useful role in analysis related to micro/nano sized structures. Therefore, several researchers have applied the gradient elasticity for the mechanical analysis of the nanostructures in more recent years (Samaeia et al., 2011; Wang, 2010; Danesh et al., 2012; Shen et al., 2012).

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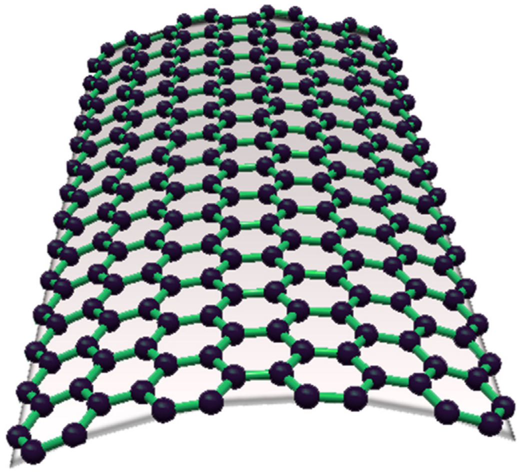

Fig. 1. Perspective view of a curved graphene ribbon (CGR).

Motivated by the above discussions, we investigate the vibrational properties of CGRs from the modified shell model for the first time in the literature. In this study, the gradient elasticity theory with two scale parameters is used to modify the classical shell model. It would appear that gradient elasticity theory could potentially play a useful role in analysis related to nanotechnology applications (Aifantis, 2009). The present work explores this potential in the context of a specific application. The displacement amplitudes are assumed to be small and so linear Novozhilov's shallow shell theory is used and the end conditions are considered simply supported at all sides of CGRs. The Novozhilov theory is known as a highly reliable theory that can be used for most shapes regardless of the size of their cross-sectional radius. The orthotropic properties of the graphene have been reported by Chang (2010). Therefore, the material properties of CGRs are assumed to be orthotropic here. The displacement form of the governing equation is developed and analytical solution is obtained. Moreover, the vibrational characteristics for both the flat and curved graphene ribbons are compared. This model is then used to study the effects of various parameters such as the radius of curvature, widths and length scale parameters on the natural frequencies of CGRs. We hope this investigation will be helpful for interesting and potential applications of CGRs in future.

2. Formulation

2.1. Gradient elasticity

The gradient elasticity theory was developed by combining Eringen stress-gradient and stable strain-gradient theory. This theory incorporates more than one length scale. The constitutive relations of the gradient elasticity can be written as follows (Askes and Aifantis, 2009):

$$(1 - l_d^2 \nabla^2)\sigma_{ij} = C_{ijkl}(1 - l_s^2 \nabla^2)\varepsilon_{kl} \quad (1)$$

where l_s is the relevant length scale for statics and l_d is the length scale that is added for using in dynamics. σ_{ij} and ε_{kl} are the components of stress and strain tensors, respectively. C_{ijkl} is assumed to represent the component of fourth-order linear elastic material tensor and ∇^2 denotes Laplacian operator. The two length scales can be related to the size of the Representative Volume Element (RVE) in statics and dynamics (Gitman et al., 2005; Bennett et al., 2007). Generally, l_s is not identical to l_d. When $l_d = 0$ the theory is a special form of Mindlin's strain gradient elasticity and when $l_s = 0$ the theory reduces to Eringen's stress gradient elasticity theory. In addition, the gradient elasticity in the vanishing limit of l_d, l_s reverts to classical elasticity, which can be seen by letting $l_d, l_s \to 0$ in Eq. (1), to obtain generalized Hook's law of classical elasticity.

2.2. Orthotropic elastic shell model for CGRs

CGR is considered as a circular cylindrical panel with an equivalent thickness h and mean radius R (Fig. 2). The panel is assumed to be homogeneous but orthotropic. It is assumed that the initial stress due to the curvature of CGR is relaxed. A curvilinear coordinate system (x_1, x_2, x_3) is considered. The displacements of an arbitrary point of coordinate (x_1, x_2) on the middle surface of CGR are denoted by u, v and w, in the x_1, x_2 and x_3 directions, respectively. The strain components ε_{11}, ε_{22} and ε_{66} at an arbitrary point of CGR are related to the middle surface strains e_1, e_2 and e_6 and to the changes in the curvature and torsion of the middle surface k_1, k_2 and k_6 by the following three relationships

$$\varepsilon_{11} = e_1 + x_3 k_1 \quad \varepsilon_{22} = e_2 + x_3 k_2 \quad \varepsilon_{66} = e_6 + x_3 k_6 \quad (2)$$