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Modeling of unidirectional fibre-reinforced composites under fibre damage

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ABSTRACT

Of interest in this work is the description of unidirectional fibre-reinforced composites where special emphasis is placed on the fibre breakage damage mode. A simple, but efficient, yield concept is adopted within the continuum damage mechanics framework where damage flow is directly linked to the strain history along the direction of the fibres. The modeling is embedded into a formulation of transverse isotropy that keeps the fibre-damage modeling unchanged when coupled to other phenomena that solely affect the pure shear part of the behavior. In fact, it is mostly observed that creep in fibre-reinforced composites is essentially due to the matrix constituent whose role is to deform and support stresses primarily in shear. This specific example is detailed in the present paper for illustrative purposes where, among others, the occurrence of tertiary creep is made possible to predict. On the numerical side, the algorithmic design is developed for a straightforward implementation within the context of the finite element method.

1. Introduction

Composite materials belong to a very important class of materials which are employed in a wide range of industrial applications and, recently, are gaining more interest in civil engineering conceptions. The particular case of fibre-reinforced composites consists of fabric structures where fibres are continuously arranged in a matrix. As a consequence, the characteristic macroscopic behavior exhibits strong directional dependencies.

Damage can significantly reduce structural stiffness before eventual catastrophic failure. It is then of interest to build predictive tools within an appropriate modeling framework to ensure maximum security and serviceability of the structures. During the recent decades, an extensive research activity has been noticed on these lines. For instance, various micromechanical models have been proposed in the literature that consider the mechanical behavior of the composites constituents, *i.e.* the fibre, the matrix and their interaction, see for example Carvelli and Taliercio (1999), Ohno et al. (2002) and Okabe et al. (2005), see also the review paper by Herakovich (2012). On the other hand, phenomenological models have also been proposed that are mostly based on the concept of continuum damage mechanics, see for example Matzenmiller et al. (1995) and Voyiadjis and Deliktas (2000) among many others. The latter approach is followed in the present paper as well.

Attention is devoted in this work to unidirectional fibre-reinforced composites. The material is then transversely isotropic with respect to the single privileged direction of the fibres, and the anisotropy is here captured by a strain energy expressed in terms of the so-called integrity-basis as proposed by Spencer (1984). This basis consists of invariants of the strain tensor together with invariants of tensor products of the strain with the structural tensor, the latter being the dyadic product of the above fibres direction. It is worth mentioning that the formalism of the integrity basis has otherwise been widely employed in the finite strain range, among others, see for example Kaliske (2000), Reese (2003), Klinkel et al. (2005), Sansour et al. (2006), Merodio and Goicolea (2007) and Nedjar (2007).

Remaining in the context of the small strain theory, the five Lamé-like elastic moduli will be degraded with damage accumulation. A fibre-breakage damage mode is considered in this paper which is directly linked to the strain history along the direction of the fibres. Additional damage phenomena such as, debonding are out of the scope of this paper. They will be the object of future works. The failure of the composite matrix is also taken into account as the evolution of damage affects the whole set of the elastic constants. Among others, we discuss the choice of a simple and efficient yield criterion together with the companion constrained flow rule that controls the damage evolution. In particular, we consider that, along the fibres, damage occurs only in tension and no damage takes place in compression.

On another hand, it is well known that long-term as well as short-term matrix creep can occur. Matrices experience creep

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primarily due to shearing. It is then of importance to take into account this fact in an unified modeling framework. Within the aforementioned formulation, this behavior has recently been captured in Nedjar (2011). As shown, the constitutive law can be decomposed into fibre-directional, transverse, and pure shear parts. Viscoelasticity is then introduced such that it solely affects the pure shear part of the behavior. In parallel to the theoretical developments, the numerical integrations of the whole sets of local evolution equations are addressed throughout the paper for a straightforward implementation within the finite element method.

Notation: Throughout the paper, bold face characters refer to second- or fourth-order tensorial quantities. In particular, $\mathbf{1}$ denotes the second-order identity tensor with components δ_{ij} (δ_{ij} being the Kronecker delta), and $\boldsymbol{I}$ is the fourth-order unit tensor of components $I_{ijkl} = 1/2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$. The double dot symbol ':' is used for double tensor contraction, *i.e.* for any second order tensors $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{A} : \boldsymbol{B} = \mathrm{tr}[\,\boldsymbol{A}\boldsymbol{B}^T] = A_{ij}B_{ij}$ where, unless specified, summation on repeated indices is always assumed. One has the property $\mathrm{tr}[(\cdot)] = [(\cdot) : \mathbf{1}]$ for the trace operator "tr". The notation $\otimes$ stands for the tensorial product, *i.e.* $(\boldsymbol{A} \otimes \boldsymbol{B})_{ijkl} = \boldsymbol{A}_{ij}\,\boldsymbol{B}_{kl}$. For any two vectors $\vec{U}$ and $\vec{V}$, the second-order tensor $\vec{U} \otimes \vec{V}$ is of components $(\vec{U} \otimes \vec{V})_{ij} = U_iV_j$. Furthermore, the dot operator $(\dot{\cdot})$ always refers to the time derivative.

2. Linear elasticity formulation

In all what follows, we denote by $\vec{V}$ the *unit* vector that characterizes the fibre direction of a one-family fibre-reinforced composite. This vector of components V_i ($i = 1, 2, 3$) with respect to a fixed global Cartesian basis, say $\{\vec{e}_i\}_{i=1,2,3}$, is regarded as a continuous function of the position. The fibre's direction is not necessarily the same at each point and, hence, the material is in fact *locally* transversely isotropic with respect to this single preferred direction. In the same way, we also introduce the continuous tensor field of the micro-structure defined by the dyadic product $\boldsymbol{M} = \vec{V} \otimes \vec{V}$. Notice the useful property $\boldsymbol{M}^n = \boldsymbol{M}$ for any integer $n > 0$, *i.e.* $\boldsymbol{M}$ is idempotent.

In this work, the anisotropy is captured by the integrity-basis formulation as introduced by Spencer (1984). Briefly, the strain energy, that is denoted here by $\mathcal{W}$, is expressed in terms of invariants. For the particular case with one family of fibres, the basis of three characteristic quantities of isotropy extends to five irreducible invariants

$$I_1 = \mathrm{tr}[\boldsymbol{\varepsilon}], \quad I_2 = \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}, \quad I_3 = \det[\boldsymbol{\varepsilon}], \quad I_4 = \boldsymbol{\varepsilon} : \boldsymbol{M}, \quad I_5 = \boldsymbol{\varepsilon}^2 : \boldsymbol{M} \tag{1}$$

where $\boldsymbol{\varepsilon}$ is the infinitesimal strain tensor, $\det[\cdot]$ designating the determinant operator.

Within the linear theory, the strain energy is quadratic with respect to the strain tensor $\boldsymbol{\varepsilon}$ and, hence, independent of the cubic invariant I_3, *i.e.* $\mathcal{W} \equiv \mathcal{W}(I_1, I_2, I_4, I_5)$. Its most elegant expression is given by,

$$\mathcal{W} = \frac{1}{2}\lambda I_1^2 + \mu_T I_2 + \alpha I_1 I_4 + 2(\mu_L - \mu_T) I_5 + \frac{1}{2}\beta I_4^2 \tag{2}$$

where the five independent material parameters λ, μ_T, μ_L, α and β are Lamé-like elastic constants, see also Holzapfel (2000), Kaliske (2000) and Nedjar (2011) for details. They are related to the standard engineering parameters as

$$\begin{aligned}
\mu_L &= G_{LT}\\
\mu_T &= \frac{E_T}{2(1+\nu)}\\
\lambda &= \frac{\nu E_L + \nu_{LT}^2 E_T}{E_L/E_T(1-\nu^2) - 2\nu_{LT}^2(1+\nu)}\\
\alpha &= \frac{(\nu_{LT} + \nu\nu_{LT} - \nu)E_L - \nu_{LT}^2 E_T}{E_L/E_T(1-\nu^2) - 2\nu_{LT}^2(1+\nu)}\\
\beta &= \frac{(1-\nu^2)E_L^2 + \nu_{LT}^2 E_T^2 + (\nu - 2\nu_{LT}(1+\nu))E_L E_T}{E_L(1-\nu^2) - 2E_T\nu_{LT}^2(1+\nu)} - 4G_{LT} + \frac{E_T}{1+\nu}
\end{aligned} \tag{3}$$

where the subscript L refers to the fibres' direction, and T to the transverse plane normal to it.

The stress tensor $\boldsymbol{\sigma}$ being given by the state law $\boldsymbol{\sigma} = \partial\mathcal{W}/\partial\boldsymbol{\varepsilon}$, use of the following useful results within the derivation employing the chain rule

$$\frac{\partial I_1}{\partial \boldsymbol{\varepsilon}} = \mathbf{1}, \quad \frac{\partial I_2}{\partial \boldsymbol{\varepsilon}} = 2\boldsymbol{\varepsilon}, \quad \frac{\partial I_4}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{M}, \quad \frac{\partial I_5}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{M}\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}\boldsymbol{M} \tag{4}$$

leads to the constitutive relation given by

$$\begin{aligned}
\boldsymbol{\sigma} = {}& \lambda\,\mathrm{tr}[\boldsymbol{\varepsilon}]\,\mathbf{1} + \alpha\left\{[\boldsymbol{\varepsilon} : \boldsymbol{M}]\mathbf{1} + \mathrm{tr}[\boldsymbol{\varepsilon}]\boldsymbol{M}\right\} + \beta[\boldsymbol{\varepsilon} : \boldsymbol{M}]\boldsymbol{M} + 2\mu_T\,\boldsymbol{\varepsilon}\\
& + 2(\mu_L - \mu_T)\left\{\boldsymbol{M}\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}\boldsymbol{M}\right\}
\end{aligned} \tag{5}$$

In this form, there is no need to select a coordinate system $\{\vec{e}_i\}_{i=1,2,3}$ such that one of the coordinate axes coincide with the direction of the fibres.

3. Modeling of fibre breakage

In order to provide a tool for structural simulations, a phenomenological modeling at the macroscale is of interest. We choose for this an internal variable model within the context of the nowadays well-known continuum damage mechanics, see for instance Lemaitre and Chaboche (1994). In this spirit, the residual elastic properties of the material are reduced with growing damage. The model adopted in this work is described in the following together with an algorithmic setting for its numerical implementation.

3.1. A strain-based fibre breakage damage model

The elastic-damage law we choose constitutes the simplest form where damage is coupled to elasticity. We write

$$\boldsymbol{\sigma} = (1-d)\boldsymbol{\sigma}_0 \tag{6}$$

where $\boldsymbol{\sigma}_0$ is the effective undamaged stress tensor given by Eq. (5). The newly introduced scalar d is the internal fibre damage variable with value 0 when the fibres are undamaged and 1 when they are completely damaged. The way this damage evolves is a matter of modeling and, in all cases, it is necessary to specify complementary equations.

At the local level, damage must be linked to the strain along the fibres. This latter, being given by the strain projection $\vec{V} \cdot \boldsymbol{\varepsilon}\vec{V}$, is no more than the invariant $I_4 = [\boldsymbol{\varepsilon} : \boldsymbol{M}]$ defined in Eq. (1)_4. Therefore, I_4 constitutes an excellent candidate to govern this damage mode. We choose in this work the following criterion

$$\mathcal{F}(d; I_4) = (1-d)^m I_4 - \varepsilon_F \le 0 \tag{7}$$

where $\varepsilon_F > 0$ is the strain-like initial damage threshold, and the constant parameter $m > 0$ controls the hardening/softening response as shown below. This criterion simply means that damage evolves when the strain along the fibres reaches the value $\varepsilon_F/(1-d)^m$. In