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Band gaps of Lamb waves propagating in one-dimensional periodic and nesting Fibonacci superlattices thin plates

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ABSTRACT

We study the band-gap structures of Lamb waves propagating in one-dimensional periodic sequences thin plate and nesting Fibonacci superlattices thin plates with different generation numbers. The dispersion curves are calculated based on the plane wave expansion method. It is found that more band gaps occur in nesting Fibonacci superlattices than in periodic phononic crystals. This is because the cells in nesting Fibonacci superlattices are quasi-periodic sequences at micro-scale, which can cause splitting of band gaps. Additionally, the periodic feature of nesting Fibonacci superlattices at macro-scale enhances Bragg scattering, which causes band gaps to become flat. As the growth of generation numbers, the characteristic of curves becomes flatter. Compared with the periodic model, the special structures of nesting Fibonacci superlattices can be used to adjust the width of band gaps and the frequency ranges of phononic crystals.

1. Introduction

There has been a growing interest in using the Lamb waves for a variety of physical, chemical and biological sensors. Due to the coupling of longitudinal and transversal strain components at the plate boundaries with wave vectors (Lamb waves), the elastic waves in the composite plate are more interesting and challenging than that of bulk and surface acoustic waves [1–4]. Chen et al. [5] have investigated the band-gap structures of lower-order Lamb waves in one-dimensional (1D) periodic composite thin plates according to a rigorous theory of elastic waves, and the band gaps for low-order Lamb wave modes in systems made of alternating strips of Tungsten materials and Silicon resin have been demonstrated. Zhang et al. [6] have reported a type of phononic crystals (PCs) manufactured by patterning periodical air-filled holes into thin plates, and they confirmed the existence of band gaps in the PCs through laser ultrasonic measurement. Hsu et al. [7] have studied the propagation of Lamb waves in two-dimensional PC plates, based on Mindlin's plate theory and the plane wave expansion (PWE) method. J.O. Vasseur et al. [8] have introduced a supercell plane wave expansion method for the calculation of elastic band. They computed the band structure of solid–solid and air–solid two-dimensional phononic crystal plates and investigated the effect of the constituent materials, of the plate thickness, and of the geometry of the array on the band structure. Gao et al. [9] have further studied the propagation of lower-order Lamb waves in 1D PC layer coated on uniform substrate, and the influences of different substrates on band-gaps for low-order Lamb wave modes were analyzed.

We note that various modified photonic crystal structures, such as one dimension periodic superlattices where each cell is formed by loops and segments following the Fibonacci sequence, have been studied by Y. El Hassouani [10]. Different properties of surface electromagnetic waves in the periodic superlattices related to the bulk bands such as the fragmentation of the frequency spectrum and the power law were analyzed and discussed. Omnidirectional zero-$\overline{n}$ gaps in 1D photonic crystals with a Fibonacci basis containing frequency dependent metamaterials have been examined by W. J. Hsueh et al. [11]. They observed that the change in the width and location of the omnidirectional zero-$\overline{n}$ gap was limited for the Fibonacci photonic crystals with different generation orders. C. Forestiere et al. [12] have proposed a simple and effective computational approach, based on the electric quasistatic approximation, to analyze both the dipolar modes of aperiodic deterministic arrays of metal nanoparticles and their coupling with an external electric field. The method was applied to the case of linear arrays generated according to the Fibonacci sequence. The results of their investigation demonstrated the potential of the proposed computational approach for the accurate design of aperiodic plasmonic devices. It's necessary to study Lamb waves propagating in 1D nesting Fibonacci superlattices phononic crystal plates.

In this paper, we calculate the dispersion curves of Lamb waves propagating in 1D periodic sequences thin plate and the nesting Fibonacci superlattices thin plates with different generation numbers. We observe that more band gaps occur in nesting Fibonacci superlattices structures with generations S_3, S_4 and S_6. With the growth of generation numbers, it is found that the characteristic of curves becomes flatter. This is because the cells in nesting Fibonacci superlattices are quasi-periodic sequences at micro-scale, which can cause split-up of

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band gaps. Additionally, the feature of periodicity at macro-scale enhances Bragg scattering, which causes that band gaps become flat. We can make the selection of a specific arrangement according to the request for the position and width of band gaps. It provides flexible choices for real engineering requirement.

2. Periodic and nesting Fibonacci structures

Fibonacci sequence can be realized experimentally by arranging the two building blocks A and B in such a way that the nth stage of the process S_n is given by the recursive rule $S_n = S_{n\text{-}1}S_{n\text{-}2}$, for $n \geq 2$, starting with $S_0 = A$ and $S_1 = B$. In this way, Fibonacci generations are $S_0 = A$; $S_1 = B$; $S_2 = BA$; $S_3 = BAB$; $S_4 = BABBA$; $S_5 = BABBABAB$; $S_6 = BABBABABBABBA$, etc. As blocks A with width d_A and blocks B with width d_B are alternately put along a chain the periodic structure is obtained, as shown in Fig. 1a. The thickness of the plate is L.

In nesting Fibonacci superlattices, the array of A and B in each cell follows the Fibonacci generations. Fig. 1b shows the configuration of the composite thin plate for nesting Fibonacci superlattices with generation number S_4. This lattice structure is combined by cells and each cell is formed by Fibonacci sequence with the generation number S_4. In this paper, the nesting Fibonacci superlattices models with generation numbers S_3, S_4 and S_6 are chosen to compare the band-gap features of Lamb waves with the periodic model.

3. Formulation of PWE method

We assume that Lamb waves propagate along the x direction, and the thin plate is bounded by planes $z = 0$ and $z = L$. We consider a two-dimensional problem, in which all field components are assumed to be independent of the y direction. In an inhomogeneous linear elastic medium with no body force, the equation of motion for displacement vector $u(x,z,t)$ can be written as:

$$\rho(x)\ddot{u}_p = \partial_q\left[c_{pqmn}(x)\partial_n u_m\right] \quad (p = 1,2,3) \tag{1}$$

Where $\rho(x)$ and $c_{pqmn}(x)$ are the mass density and elastic stiffness, respectively. For the spatial periodicity in the x direction, the material constants, $\rho(x)$ and $c_{pqmn}(x)$ can be expanded into the Fourier series form:

$$\rho(x) = \sum_G exp(iGx)\rho_G \tag{2}$$

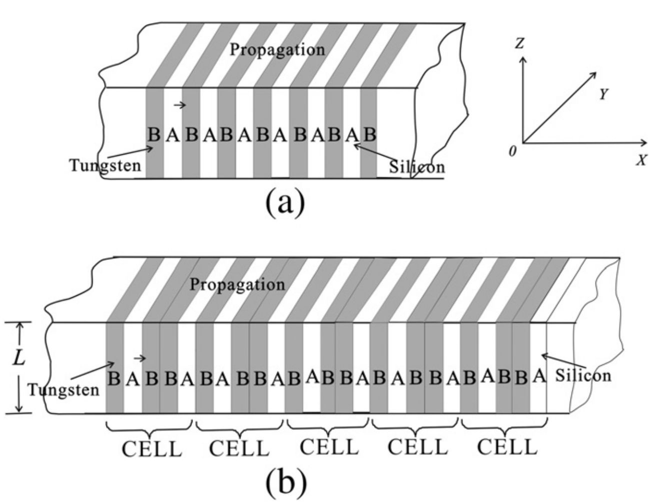

Fig. 1. Configuration of the 1D composite plates consists of trips A and B (a — periodic model; b — nesting Fibonacci superlattices model with generation number S_4). Materials in A and B are silicon and tungsten, respectively. The thickness of thin plates is L.

$$c_{pqmn}(x) = \sum_G exp(iGx)c_G^{pqmn} \tag{3}$$

The subscript G is the 1D reciprocal-lattice vectors (RLVs). According to the Bloch theorem, by expanding the displacement vector $u(x,z,t)$ into the Fourier series, one obtains:

$$u(x,z,t) = \sum_{G'} exp(ik_x x - i\omega t)\left[exp\left(iG'x\right)U_{G'}exp(ik_z z)\right] \tag{4}$$

Where k_x is the *Bloch* wave-vector, ω is the circular frequency, $U_{G'} = \left(U^1_{G'}, U^2_{G'}, U^3_{G'}\right)$ is the amplitude of the partial waves, and k_z is the wave number of the partial waves along the z direction.

Substituting Eqs. (2)–(4) into Eq. (1), let $G_0 = G + G'$, one obtains:

$$\begin{pmatrix} M^{(1)}_{G_0,G'} + k_z^2 N^{(1)}_{G_0,G'} & k_z L^{(1)}_{G_0,G'} \\ k_z L^{(2)}_{G_0,G'} & M^{(2)}_{G_0,G'} + k_z^2 N^{(2)}_{G_0,G'} \end{pmatrix} \begin{pmatrix} U^1_{G'} \\ U^3_{G'} \end{pmatrix} = 0 \tag{5}$$

Where

$$M^{(1)}_{G_0,G'} = c^{11}_{G_0-G'}(k_x + G)\left(k_x + G'\right) - \rho_{G_0-G'}\omega^2 \tag{6}$$

$$M^{(2)}_{G_0,G'} = c^{44}_{G_0-G'}(k_x + G)\left(k_x + G'\right) - \rho_{G_0-G'}\omega^2 \tag{7}$$

$$N^{(1)}_{G_0,G'} = c^{44}_{G_0-G'} \tag{8}$$

$$N^{(2)}_{G_0,G'} = c^{11}_{G_0-G'} \tag{9}$$

$$L^{(1)}_{G_0,G'} = c^{12}_{G_0-G'}(k_x + G) + c^{44}_{G_0-G'}\left(k_x + G'\right) \tag{10}$$

$$L^{(2)}_{G_0,G'} = c^{12}_{G_0-G'}\left(k_x + G'\right) + c^{44}_{G_0-G'}(k_x + G) \tag{11}$$

Further, we transform Eq. (5) into another form:

$$\left(P_{G_0,G'}k_z^2 + Q_{G_0,G'}k_z + R_{G_0,G'}\right)U'_{G'} = 0 \tag{12}$$

Where $U'_{G'} = \left(U^1_{G'}, U^3_{G'}\right)$, and the coefficients matrix $P_{G_0,G'}$, $Q_{G_0,G'}$, $R_{G_0,G'}$ are:

$$P_{G_0,G'} = \begin{pmatrix} N^{(1)}_{G_0,G'} & \\ & N^{(2)}_{G_0,G'} \end{pmatrix} \tag{13}$$

$$Q_{G_0,G'} = \begin{pmatrix} & L^{(1)}_{G_0,G'} \\ L^{(2)}_{G_0,G'} & \end{pmatrix} \tag{14}$$

$$R_{G_0,G'} = \begin{pmatrix} M^{(1)}_{G_0,G'} & \\ & M^{(2)}_{G_0,G'} \end{pmatrix} \tag{15}$$

Eq. (12) is a generalized eigenvalue equation with respect to k_z. If we truncate the expansion of Eqs. (2)–(3) by including n RLVs, Eq. (12) generates $4n$ eigenvalues when k_x and ω are given values. For the Lamb waves, all of the $4n$ eigenvalues $k_z^{(l)}(l = 1 - 4n)$ must be included. Thus, the displacement vector of the Lamb waves can be written in the form:

$$\begin{aligned} u(x,z,t) &= \sum_{G'}{}' exp\left[i\left(k_x + G'\right)x - i\omega t\right]\left[\sum_{l=1}^{4n} U'_{G'} \exp\left(ik_z^{(l)}z\right)\right] \\ &= \sum_{G'}{}' exp\left[i\left(k_x + G'\right)x - i\omega t\right]\left[\sum_{l=1}^{4n} X^{(l)}\varepsilon^{(l)} \exp\left(ik_z^{(l)}z\right)\right] \end{aligned} \tag{16}$$