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Failure and debonding of thin circular and square tiles (islands) bonded with a compliant interlayer

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ABSTRACT

This paper presents analytical solutions for the stresses in circular thin films bonded to a substrate with a thin compliant interlayer. The axisymmetric results are shown to be an excellent approximation for square tiles (islands), provided one defines an effective diameter equal to the average of the square's diagonal and width. An analytical result is also presented for the energy release rate associated with convergent circular delamination cracks (from the outer edges of the tile inwards). These solutions are used to generate regime maps that indicate active failure mechanisms (tile yielding, interlayer yielding and delamination) as a function of constituent properties and tile size. These regime maps clearly indicate acceptable tile sizes and/or the required material properties to avoid all modes of failure.

1. Introduction

A wide variety of technologies involve the use of tile-like structures (often referred to as "islands") bonded to substrates, as shown schematically in Fig. 1. Examples include sensor arrays, displays, microelectronic packaging and thermal protection systems (e.g. [1–10,12–14]). Interlayers are commonly present between the tiles and the substrate to promote adhesion and/or to provide thermal or electrical insulation. The relationship between stresses arising from thermal expansion mismatch and tile size plays a critical role in design, as it ultimately governs the susceptibility of the system to failure by yielding, cracking or interfacial debonding (e.g. [11,13,12,14,15]).

As is well-known from shear lag theory [16], the in-plane direct stress in a tile due to a misfit strain increases from the outer edge towards the center, a consequence of the shear transfer between the tile and the underlying structure (e.g. [16–25]). (Note that many of these and other references address the problem of multiple cracking in blanket films, which leads to thin strips of finite dimension; the crack spacing dictates the tile or island size.) The peak stress at the tile center depends on the tile size relative to a characteristic shear transfer length, and asymptotically approaches the blanket-film result in the limit of large tile sizes. Even for applications where finite-sized features are not a prerequisite (e.g. a thermal protection system that has no inherent constraint on planar dimension), the stresses in large tiles may be too high to avoid failure. In such scenarios, the use of finite-sized tiles can be an effective way to reduce stresses and improve reliability (e.g. [11,13,12,14,15]).

Hence, a central design variable for such systems is the tile size. A maximum allowable size might be prescribed in order to avoid failure, given a pre-determined set of thermomechanical properties for the constituents. Alternatively, if the tile size is fixed by other considerations (e.g. sensor area), one might pose the question in terms of an acceptable range of properties (such as adhesion or coefficient of thermal expansion (CTE) mismatch). In such design exercises, closed-form relationships between geometry, properties and stress are highly advantageous, in that they eliminate the need for cumbersome numerical studies of the parameter space. This is particularly true for applications in which material selection is part of the design process (as opposed to being fixed a priori), since there are likely many possible combinations of materials and tile sizes that are acceptable.

Here, we present closed-form solutions for deformations and stresses in a thin circular tile mounted on a thick substrate via a compliant interlayer. The solutions are shown to be accurate approximations for square tiles, subject to a suitable definition for the effective tile diameter. In turn, the stress solutions are used to estimate the steady-state energy release rates for interlayer debonding. The steady-state energy release rate corresponds to the maximum possible driving force, obtained when the crack length is much greater than the tile thickness (e.g. [26,27] and references therein). Previous calculations have demonstrated that the energy release rate grows quickly as a function of crack length, reaching steady state for lengths (measured from the outer edge

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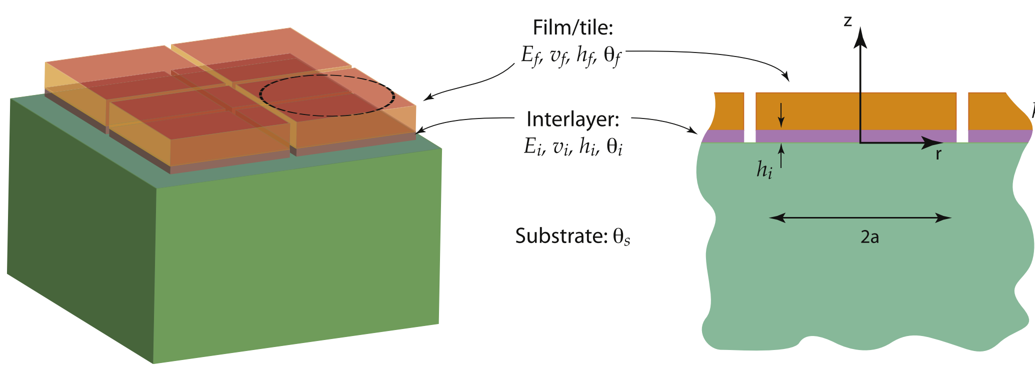

Fig. 1. Schematic of the tiling system. The problem is analyzed as axisymmetric, with the tile (island) diameter denoted as *a*. It is shown via comparison with finite element analysis (FEA) of square tiles that the axisymmetric model is accurate provided the effective radius is taken $a=(1/4)\left(w+\sqrt{2}w\right)$, where *w* is the width of the square tile.

of the tile) on the order of several tile thicknesses [25–27]. The resulting solutions for stresses and energy release rates are then used to construct regime maps that facilitate design of tiling systems. The maps depict various failure modes (yielding, cracking or interface debonding) in terms of system geometry (characterized by layer thicknesses and tile size) and material properties (e.g. stiffness, yield strength, thermal expansion and toughness).

Though this analysis is inspired by and resembles a variety of previous shear-lag analyses of stress and debonding in finite-sized features [16–25,28–31], a critical distinguishing feature of the present work is the treatment of the in-plane tile stresses acting parallel to the free edges. Here, these stresses are non-zero and dictated by the tile size. Previous treatments that assume plane-strain deformation (i.e. a semi-infinite strip) or purely biaxial deformation lead to inappropriate predictions of direct stress in the direction parallel to the free edge. That is, if one assumes purely biaxial stress and imposes the condition that the in-plane stresses are zero at the free edge, then the stress parallel to the free edge is assumed to be zero, which is not the case. Similarly, if one assumes plane-strain deformation of a semi-infinite strip, then the stress parallel to the free edge is not a function of tile size, which is not the case. The present model properly imposes the condition that the stress along the edges is zero in the direction normal to the surface, and dictated by the tile size in the direction parallel to the edge. Further, the model enables failure maps that indicate transitions in failure mechanism as a function of tile size and key dimensionless parameters identified here. As with all shear-lag analyses, the model assumes that displacements occur only in the plane of the tile, such that through-thickness effects are negligible. In order for this to be valid, the aspect ratio of the tiles (planar dimension divided by the total thickness) must be large. It is further assumed that the tiling system is attached to a substrate that is much thicker than the top layers, such that bending in the multilayer is negligible.

2. Model and results

The constituents are assumed to be linearly elastic with the properties: *E*-Young's modulus, *ν*-Poisson's ratio and *h*-thickness. The substrate is assumed to be semi-infinite, such that bending deformation is negligible. This implies that the stress in the tile scales with the misfit strain given by θ_s–θ_f, where $\theta_{s,f}$ are the eigenstrains in the substrate and film (top tile). (For example, for thermal misfit, $\theta=\alpha\Delta T$, where α—CTE, and $\Delta T=T-T^o$ is the temperature change relative to the stress-free reference temperature T^o). Subscripts refer to a specific layer: *f*-film (or tile), *s*-substrate, and *i*-interlayer. The analysis assumes that the majority of deformation in the tile is axisymmetric, with only radial displacements being non-zero. It is demonstrated that this is an accurate approximation for square tiles, with minor deviations near the tile corners that are not likely to impact design choices.

2.1. Shear lag analysis and displacement solution

The model assumes only radial displacements, $u(r)$, such that the kinematic and constitutive relationships for the film are given by:

$$\epsilon_r=\frac{\partial u(r)}{\partial r};\ \frac{\left(1-\nu_f^2\right)\sigma_r^f}{E_f}=\epsilon_r+\nu_f\epsilon_\theta-\left(1+\nu_f\right)\theta_f \tag{1}$$

$$\epsilon_\theta=\frac{u(r)}{r};\ \frac{\left(1-\nu_f^2\right)\sigma_\theta^f}{E_f}=\epsilon_\theta+\nu_f\epsilon_r-\left(1+\nu_f\right)\theta_f. \tag{2}$$

Equilibrium in the film dictates the following:

$$\frac{\partial\sigma_r^F}{\partial r}+\frac{\sigma_r^f-\sigma_\theta^f}{r}+\frac{\partial\sigma_{rz}^f}{\partial z}=0. \tag{3}$$

In the present approximation, the shear stress in the thin interlayer, σ_{rz}^i, is assumed to be uniform through its thickness, and governed by the difference of the displacements at the top and bottom of the interlayer:

$$\sigma_{rz}^i=\frac{E_i}{2(1+\nu_i)}\left(\frac{u(r)-\theta_s r}{h_i}\right), \tag{4}$$

where $u(r)$ is the displacement of the top of the interlayer, which is equal to the film displacement. The quantity $\theta_s r$ reflects the uniform outward expansion of the bottom of the interlayer due to the substrate's expansion. In this regard, the effect of mechanical stretching of the substrate can be easily accounted for by including the imposed strain in the definition of θ_s, as in $\theta_s=\alpha_s\Delta T_s+\epsilon_a$, where ϵ_a is the strain applied to the substrate. The shear stress in the film at the interface acts opposite to that in the interlayer (as defined above); further, the shear stress is zero at the top of the film. Assuming that the film is thin, the gradient of shear stress in the film is well-approximated by:

$$\frac{\partial\sigma_{rz}^f}{\partial z}\approx-\frac{\sigma_{rz}^i}{h_f}=\frac{E_i}{2(1+\nu_i)}\left(\frac{u(r)-\theta_s r}{h_i h_f}\right). \tag{5}$$

The governing equation for radial displacements is obtained by combining Eqs. (1)–(5). Using the normalizations $u=a\cdot\bar{u}$ and $r=a\cdot\bar{r}$, where *a* is the tile radius, one obtains the following governing equation:

$$\bar{u}''+\frac{\bar{u}'}{\bar{r}'}-\frac{\bar{u}}{\bar{r}^2}-\lambda^2\left(\bar{u}-\theta_s\cdot\bar{r}\right)=0 \tag{6}$$