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An approximate solution of the interaction between an edge dislocation and an inclusion of arbitrary shape

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Abstract

An approximate solution of the interaction force between an edge dislocation and an inclusion of arbitrary shape is derived, from which a set of succinct formulas for several special inclusion shapes are obtained. As compared with several classical solutions to special inclusion shapes, the present approximate solution has fairly good accuracy.

Keywords: Edge dislocation; Inclusion; Transformation strain

1. Introduction

The study of interaction between dislocation and inclusion has received considerable interest over years because it is important for understanding the mechanical behavior of many materials. The study is traditionally based on solution of appropriate boundary value problems in the linear theory of elasticity and, in only a few ideal cases, have analytical solutions been obtained, such as circular inclusion (Dundurs and Mura, 1964; Dundurs and Gangadharan, 1967; Luo and Chen, 1991; Xiao and Chen, 2000), elliptical inclusion (Stagni and Lizzio, 1983; Santare and Keer, 1986; Yen and Hwu, 1994), lamellar inclusion (Chou, 1966) and a surface layer (Weeks et al., 1968).

An exactly analytical solution of the interaction between a dislocation and an inclusion of arbitrary shape is very difficult, even impossible, to obtain based on linear theory of elasticity. Hence, an approximate estimation is desirable in practice. In our previous studies (Li and Shi, 2002), an approximate solution to the interaction of a screw dislocation with an inclusion of arbitrary shape has been obtained based on Eshelby equivalent inclusion theory. However, this solution was obtained based on the assumption that Poisson's ratios of the inclusion and the matrix material are the same. This assumption is correct for screw dislocation, because Poisson's ratio has no effect on the solution. Unfortunately, there is a strong effect of the Poisson's ratio on the interaction between an edge dislocation and an inclusion (Dundurs and Mura, 1964).

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In the present study, an approach to determine the interaction between an edge dislocation and an inclusion of arbitrary shape is proposed. A general approximate solution to determine the force acting on the dislocation is obtained, from which a set of simple formulas for several special inclusion shapes are included. It is shown that, in comparison with corresponding classical solution, the present solution has fairly good accuracy.

2. Model and formulation

Fig. 1 shows an inclusion of arbitrary shape within a straight edge dislocation stress field, σ_{ij}^{d}. The inclusion will undergo a transformation strain, e_{ij}^{T}, due to inhomogeneity between matrix material and the inclusion.

To predict the crack-tip shielding effect induced by modulus reduction in the micro-cracking zone around main crack-tip in brittle solids, Hutchinson (1987) calculated the transformation strain, which results in the shielding effect, from first order effect of the modulus difference. Recently, Hutchinson's approach was used by Yang and Li (2003) to predict the interaction between model I crack and an inclusion of arbitrary shape. Here, we use the same techniques to calculate the transformation strain of an inclusion induced by an edge dislocation stress field.

Let $\overline{M}$ and M be the compliance of the isotropic materials inside and outside the inclusion, then the strains in the inclusion are approximately given by

$$\varepsilon_{ij} \approx \overline{M}_{ijkl}\sigma_{kl}^{\mathrm{d}} = M_{ijkl}\sigma_{kl}^{\mathrm{d}} + (\overline{M}_{ijkl} - M_{ijkl})\sigma_{kl}^{\mathrm{d}} \tag{1}$$

With the identification

$$e_{ij}^{\mathrm{T}} = (\overline{M}_{ijkl} - M_{ijkl})\sigma_{kl}^{\mathrm{d}} = \widetilde{M}_{ijkl}\sigma_{kl}^{\mathrm{d}} \tag{2}$$

Using the well-known expressions of $\overline{M}$ and M for isotropic elasticity, the non-zero components of the compliance matrix $\widetilde{M}$ are as follows:

$$\left.\begin{aligned} &\widetilde{M}_{1111} = \widetilde{M}_{2222} = \widetilde{M}_{3333} = \frac{1}{E_{\mathrm{i}}} - \frac{1}{E_{\mathrm{m}}} \\ &\widetilde{M}_{1122} = \widetilde{M}_{2211} = \widetilde{M}_{1133} = \widetilde{M}_{3311} = \widetilde{M}_{2233} = \widetilde{M}_{3322} = -\frac{\nu_{\mathrm{i}}}{E_{\mathrm{i}}} + \frac{\nu_{\mathrm{m}}}{E_{\mathrm{m}}} \\ &\widetilde{M}_{4444} = \widetilde{M}_{5555} = \widetilde{M}_{6666} = \frac{1+\nu_{\mathrm{i}}}{E_{\mathrm{i}}} - \frac{1+\nu_{\mathrm{m}}}{E_{\mathrm{m}}} \end{aligned}\right\} \tag{3}$$

where E_{i}, ν_{i} and E_{m}, ν_{m} are Young's modulus and Poisson's ratio of the inclusion and matrix material, respectively.

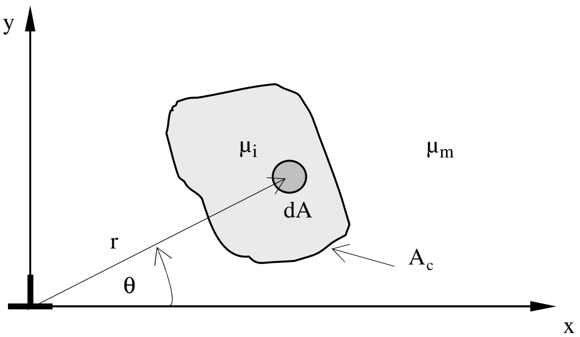

Fig. 1. An edge dislocation near an inclusion of arbitrary shape.