Available online at www.sciencedirect.com

SCIENCE DIRECT®

Mechanics Research Communications 33 (2006) 818–829

MECHANICS RESEARCH COMMUNICATIONS

www.elsevier.com/locate/mechrescom

New control laws for stabilization of a rigid body motion using rotors system

Awad El-Gohary *

Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

Available online 27 April 2006

Abstract

This paper presents a new class of globally asymptotic stabilizing control laws for dynamics and kinematics attitude motion of a rotating rigid body. The rigid body motion is controlled with the help of a rotor system with internal friction. The Lyapunov technique is used to prove the global asymptotic properties of the stabilizing control laws. The obtained control laws are given as functions of the angular velocity, Cayley–Rodrigues and Modified-Rodrigues parameters. It is shown that linearity and nonlinearity of the control laws depend not only upon the Lyapunov function structure but also the rotors friction. Moreover, some of the results are compared with these obtained in the literature by other methods. Numerical simulation is introduced.

Keywords: Rigid body; Rotors system with friction; Global stabilization; Cayley–Rodrigues and Modified-Rodrigues parameters

1. Introduction

In recent years a considerable effort has been devoted to the design of control laws for challenging dynamical systems, such as robot manipulators, high-performance rigid spacecraft, satellite and space vehicles. The problem of description and control of the attitude motion of rigid bodies is investigated in Tsiotras (1996, 1994) and Rotea (1998).

A lot of literatures use Euler angles for the orientation of a rigid body and the other use the direction-cosine (see, for example, El-Gohary, 2005b,c, 2001b; Krementulo, 1977). Unfortunately, neither the Euler angles nor the direction-cosine is the optimal using for the kinematic attitude of a rigid body motion. Since the Euler angles have singularities in the 3–1–3 Euler angles when $\theta = 0$ or π. If instead we had used 3–1–2 Euler angles, these would have been well behaved at $\theta = 0$ or π but would have shown singular behavior when $\theta = \pm\pi/2$ (see Moore, 1994). Because of the singularity difficulty of the Euler angles and large number of singularities of

* Present address: Department of Statistics and OR, Faculty of Science King Saud University, P.O. Box 2455, Riyadh, Saudi Arabia. Fax: +9 661 4676274.

E-mail addresses: elgohary0@yahoo.com, aigohary@ksu.edu.sa

0093-6413/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.mechrescom.2006.03.008

elements in the direction-cosine matrix, we will use another representation of a rigid body kinematics attitude such as Euler parameters, Cayley–Rodrigues and Modified-Rodrigues parameters.

The results in this paper complement and extend similar results published recently in terms of Euler parameters kinematic parameterizations (El-Gohary, 2002, 2001a, 1996, 2005a; Tsiotras, 1994). Euler parameters, Cayley–Rodrigues and Modified-Rodrigues parameters, can be employed to prove global asymptotic stability of a rigid body motion without imposing conditions on the control gains and system parameters. This motivated us to use these parameters for obtaining a new class of control laws that globally asymptotically stabilize a rigid body motion about its equilibrium state.

Euler parameters were employed in El-Gohary (2002, 2001a) to study the global asymptotic stabilization of a rigid body rotational motion with the help of rotor and moving masses systems respectively, using Lyapunov technique. Moreover, these parameters also were employed in the paper (El-Gohary, 1996, 2000) for describing the rigid body programmed motion and proving the global asymptotic stabilization of the rigid body rotational motion. Also, Euler parameters are used in El-Gohary (1997) to study the global asymptotic stabilization of the relative programmed motion of a satellite-gyrostat. The Modified-Rodrigues parameters as a stereographic projections of Euler parameters are used in El-Gohary (2005c) to study the problem of global optimal stabilization of a rigid body rotational motion about the equilibrium state. The problem of controlling the rotational motion of a rigid body using three independent control torques is introduced in Tsiotras (1994) and Rotea (1998).

The current paper is organized as follows. Section 2 of the paper introduces the equations of motion of a rigid body carrying a rotor system taking into consideration the internal rotors friction. Section 3 contains some important first integral of both dynamical and kinematical equations. The asymptotic stability feedback control law is obtained as function of angular velocity and Euler parameters in Section 4. The main results of this paper are concentrated in Sections 5 and 6, since both of Cayley–Rodrigues and Modified-Rodrigues parameters are employed to study the problem of global stabilization of a rigid body motion about its equilibrium state with linear and nonlinear feedback control input using Lyapunov technique.

2. Formulation the problem and equations of motion

Consider a mechanical system S which consists of a rigid body P (platform) whose center of mass with a fixed point O and three symmetrical rotors (R) are attached to the principal axes of inertia of the system S, in such a way that the motion of the rotors does not modify the distribution of mass of the system S. To describe the motion of the system, two systems of coordinates are introduced. The first $O\xi\eta\zeta$ is an inertial system and the second $Ox_1x_2x_3$ is fixed in the body and coincides with the principal axes of the body at O.

Let ω_i and C_i $(i = 1, 2, 3)$ be the components of the rigid body angular velocity vector $\vec{\omega}$ and the principal moments of inertia of the system relative to the principal axes $Ox_1x_2x_3$, respectively. The resulting system dynamics are given by

$$\dot{\vec{G}} + \vec{\omega} \times \vec{G} = \vec{L}, \tag{2.1}$$

where $\vec{G}$ and $\vec{L}$ are the system total angular momentum and the external moments vectors referring to the body fixed frame $Ox_1x_2x_3$. The components of the system total angular momentum referring to the body principal axes $Ox_1x_2x_2$ are given by

$$G_i = C_i\omega_i + I_i\dot{\Phi}_i \quad (i = 1, 2, 3),$$

where I_i and Φ_i $(i = 1, 2, 3)$ are the ith rotor axial moment of inertia and the ith rotor angle of rotation referring to the body axes $Ox_1x_2x_2$.

Eq. (2.1) should be augmented by the equations the rotors motion relative to the body coordinates system $Ox_1x_2x_2$. That is

$$I_1(\dot{\omega}_1 + \ddot{\Phi}_1) = u_1 - \mu_1\dot{\Phi}_1 \quad (123), \tag{2.2}$$

where the symbol (123) means that the two other equations can be obtained from the given equation by cyclic permutations of the indices $1 \to 2 \to 3$, the dot denotes the differentiation with respect to time, u_i $(i = 1, 2, 3)$