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Free convection flow over a truncated cone embedded in a porous medium saturated with pure or saline water at low temperatures

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Abstract

Steady free convection boundary layer about a truncated cone embedded in a porous medium saturated with pure or saline water at low temperatures has been studied in this paper. The governing coupled partial differential equations are solved numerically using a very efficient finite-difference method. Several new parameters arise and the results are given for some specific values of these parameters. The obtained results for a Boussinesq fluid are compared with known results from the open literature and it is shown that the agreement between these results is very good.

Keywords: Truncated cone; Boundary layer; Porous medium; Cold or saline water

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1. Introduction

Convective flow in porous media has been a subject of great interest for the last several decades due to its numerous thermal engineering applications in various disciplines, such as geophysical thermal insulation, modeling of packed sphere beds, cooling of electronic systems, groundwater hydrology, petroleum reservoirs, coal combustors, ground water pollution, ceramic processes, to name just a few of these applications. Some of the most important analytical, numerical and experimental studies with such applications, which present the current state-of-the-art in the area of convective heat transfer in porous media, have been gathered in the monographs by Nield and Bejan (1999), Ingham and Pop (1998, 2002), Vafai (2000), Pop and Ingham (2001), and Bejan and Kraus (2003).

Studies of convective heat transfer in porous media have been carried out in the past using the Boussinesq approximation, namely the fluid density ρ varies linearly with temperature. However, this is inappropriate for water at low temperatures because of the extremum at about 4 °C in pure water at 1 atm. Such conditions occur commonly in porous medium, such as permeable soils flooded by cold lake or sea water, water–ice

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slurries, etc. A limited number of studies have been devoted in the past to the problem of convective boundary layer adjacent to heated or cooled bodies immersed in a porous medium saturated with cold water wherein a density extremum may arise. It should be mentioned that the buoyancy flow with an extremum may become very complicated, with local flow reversals and convective inversions. Density differences may then not be expressed as a linear function of the temperature. Ramilison and Gebhart (1980) examined the possible similarity solutions for vertical, buoyancy induced flow in a porous medium saturated with cold water. Lin and Gebhart (1986) have considered the corresponding case of a horizontal surface in a porous medium saturated with cold or saline water. Gebhart et al. (1983) obtained multiple steady state solutions for the problem considered by Lin and Gebhart (1986) using two numerical codes. A review of the convective flow in the vicinity of the maximum-density condition in water at low temperatures, along with relevant citations, is available in the survey by Kukula et al. (1987).

The present paper concerns the steady free convection boundary layer adjacent to a heated truncated cone embedded in an extensive porous medium saturated with either pure or saline water under the conditions in which a density extremum might occur. The density state equation used here is that proposed by Gebhart and Mollendorf (1977), which has been shown to be very accurate for both pure and saline water to a pressure level of 1000 bars up to 20 °C, and to 40% salinity. To the best of our knowledge, this problem has not been considered before. However, Yih (1999) made an analysis for free convection boundary layer about a truncated cone in a porous medium saturated with a Boussinesq fluid subjected to the coupled effects of thermal and mass diffusion.

2. Basic equations

Consider the steady free convection over a truncated cone (with half angle γ) embedded in a saturated porous medium filled with pure or saline water. It is assumed that the surface of the truncated cone is maintained at the constant temperature T_w, while the temperature of the ambient fluid is T_∞, where $T_w > T_\infty$. Fig. 1 shows the flow model and physical coordinate system. The governing boundary layer equations are given by, see Chamkha et al. (2004),

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

$$u = \frac{\rho_m g K}{\mu}\left[|T - T_m|^q - |T_\infty - T_m|^q\right]\cos\gamma \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

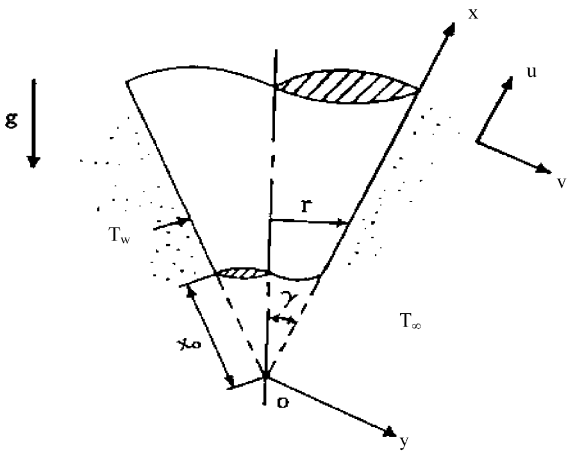

Fig. 1. Physical model and coordinate system.