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# Finite time stability analysis of PD$^{\alpha}$ fractional control of robotic time-delay systems

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**Abstract**

A finite time stability test procedure is proposed for robotic system where it appears a time delay in PD$^{\alpha}$ fractional control system. Paper extends some basic results from the area of finite time and practical stability to linear, continuous, fractional order time invariant time-delay systems given in state space form. Sufficient conditions for this kind of stability, for particular class of fractional time-delay systems are derived.


*Keywords:* Time delay; Fractional order control; Linear analysis; Finite time stability criteria; State space

## 1. Introduction

The question of stability is of main interest in control theory. Also, the problem of time-delay system has been discussed over many years. Time delay is very often encountered in different technical systems, e.g. electric, pneumatic and hydraulic networks, chemical processes, and long transmission lines. The existence of pure time delay, regardless of its presence in a control and/or state, may cause undesirable system transient response, or generally, even an instability. Numerous reports have been published on this matter, with particular emphasis on the application of Lyapunov‘s second method, or on use idea of a matrix measure (see e.g. Chen et al., 1995; Lee and Diant, 1981; Mori, 1985).

Here, another approach is presented, i.e. system stability from the non-Lyapunov point of view is considered. In practice one is not only interested in system stability (e.g. in the sense of Lypunov), but also in

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bounds of system trajectories. A system could be stable but still completely useless because it possesses undesirable transient performances. Thus, it may be useful to consider the stability of such systems with respect to certain subsets of state-space which are defined a priori in a given problem. Besides that, it is of particular significance to concern the behaviour of dynamical systems only over a finite time interval.

These boundedness properties of system responses are very important from the engineering point of view. Realizing this fact, numerous definitions of the so-called technical and practical stability were introduced such as: La Salle, Lefschet S. and Weiss, L., F. Infante (see La Salle, 1961; Weiss and Infante, 1965) have introduced various notations of stability over finite time interval for continuous-time systems and constant set trajectory bounds; Grujić (1975a,b) introduced and considered a more general type of stability (“practical stability with settling time”, practical exponential stability, etc.); Lashirer and Story (1972) introduced concept of finite-time stability, called “final stability”.

Roughly speaking, these definitions are essentially based on the predefined boundaries for the perturbation of initial conditions and allowable perturbation of system response. Thus, the analysis of these particular boundedness properties of solutions is an important step, which precedes the design of control signals, when finite time or practical stability control is concerned. Also, analysis of linear time-delay systems in the context of finite and practical stability was introduced and considered by Debeljković et al. (1989, 2001) and Lazarević et al. (2000).

Some authors are concerned with this problem from stability and robust control point of view. For a example, the effect of time-delays occurring in a proportional-integral-derivative feedback controller on the linear stability of a simple electromechanical system is investigated by analyzing the characteristic transcendental equation (Ji, 2003). The stable operation of a magnetic bearing system can only be achieved by a feedback control, where the time delays are unavoidable especially when a digital controller is used. Also, time-delay, driving through elastic belt or backlash at the driving-wheel of the motor tends to destabilize dynamical systems (Kollar et al., 2000). Unstable equilibria of mechanical systems often have to be stabilized by control force, for example, balancing of walking and standing robots. Also, problem of dynamic instability arise when there are joint/link flexibility and actuator bandwidth limit, or end-effector is in contact with a hard surface (high stiffness) to exert a desired force (Park and Chang, 2003). From the viewpoint of robot rigid dynamics, these factors may be regarded as unmodelled dynamics and disturbances on system dynamics. For robust control law authors proposed the Enhanced Time Delay Control which is a variant of Time Delay Control (Toumi and Shortlidge, 1991), which has been successfully applied to robot manipulators.

Recently, there have been some advances in control theory of fractional differential systems for stability questions (Matignon et al., 1996). However, for fractional order dynamic systems, it is difficult to evaluate the stability by simply examining its characteristic equation either by finding its dominant roots or by using other algebraic methods. At the moment, direct check of the stability of fractional order systems using polynomial criteria (e.g., Routh's or Jury's type) is not possible, because the characteristic equation of the system is, in general, not a polynomial but a pseudopolynomial function of fractional powers of the complex variables. Thus there remain only geometrical methods of complex analysis based on the so called argument principle (e.g. Nyquist type) which can be used for the stability check in the BIBO sense (bounded-input bounded-output). Also, for linear fractional differential systems of finite dimensions in state-space form, both internal and external stabilities are investigated by Matignon et al. (1996, 1998).

In this paper, particular attention is paid to the finite time stability of robotic system where a time delay appears in $PD^{\alpha}$ fractional control system. $PD^{\alpha}$ fractional time-delay control of robotic system was discussed. The problem of sufficient conditions is examined that enable system trajectories to stay within the a priori given sets for the particular class of linear fractional order time-delay systems in state-space form. To the best knowledge of author, these problems, are not yet analyzed for the fractional order time-delay systems and this class of systems.