Ultramicroscopy 185 (2018) 1–4

Contents lists available at ScienceDirect

Ultramicroscopy

journal homepage: www.elsevier.com/locate/ultramic

Variation of field enhancement factor near the emitter tip

Debabrata Biswas*, Gaurav Singh, Shreya G. Sarkar, Raghwendra Kumar

Bhabha Atomic Research Centre, Mumbai, 400 085, INDIA and Homi Bhabha National Institute, Mumbai 400 094, INDIA

ARTICLE INFO

Article history:
Received 30 May 2017
Revised 25 September 2017
Accepted 28 October 2017
Available online 31 October 2017

ABSTRACT

The field enhancement factor at the emitter tip and its variation in a close neighbourhood determines the emitter current in a Fowler–Nordheim like formulation. For an axially symmetric emitter with a smooth tip, it is shown that the variation can be accounted by a $\cos\tilde{\theta}$ factor in appropriately defined normalized co-ordinates. This is shown analytically for a hemiellipsoidal emitter and confirmed numerically for other emitter shapes with locally quadratic tips.

1. Introduction

The field of vacuum nanoelectronics involves field electron emitters with sharp tips having radius of curvature in the nanometer regime [1]. Due to the high aspect ratio, such emitters can have a large field enhancement factor, γ_a, at the apex (tip). Several models have been studied to gain insight into the dependence of height (h) and apex radius of curvature (R_a) on γ_a [2–8]. Of these, the hemiellipsoid and hyperboloid emitters are analytically tractable [9–11] while the floating sphere at plane potential has been studied extensively but its predictions ($\gamma_a \simeq h/R_a$) far exceed the known results for γ_a especially for sharp emitters [12,13]. A much studied numerical model is a cylindrical post with a hemispherical top [14] for which various fitting formulas for γ_a exist. A straightforward estimate [3] is $\gamma_a \simeq 0.7(h/R_a)$ while more elaborate ones [2–5,7] are expressed as $\gamma_a \simeq a(b + h/R_a)^{\sigma}$ with $0.9 < \sigma \leq 1$. The h/R_a dependence of γ_a can be expected for various other vertically placed emitter shapes, though there are very few concrete results.

While there is some understanding of the local field enhancement at the emitter apex, its variation in the neighbourhood of the tip is not as clear. For the hemisphere on a plane, $\gamma(\theta) = \gamma_a \cos\theta$, where $\gamma_a = 3$ and the origin is the center of the (hemi)sphere. For the hemiellipsoid or the hyperboloid, the local field at the emitter surface is known, though a geometric formula analogous to the hemisphere (the $\cos\theta$ dependence) is not known to exist. A recent numerical study [15] on the hemiellipsoid using the Ansys–Maxwell software includes the variation of γ with angle θ from the center of the ellipsoid. For a hemisphere on a cylindrical post with the origin at the center of the hemisphere, the variation with θ was reported to be quadratic [4] while another study [5] found a $\cos^{1/2}\theta$ factor to be appropriate. In both cases, the angle is measured from the centre of the sphere. For a conical emitter rounded at the apex, Spindt et al.[16] found the θ dependence (measured from the centre of curvature at the tip) to be small close to the apex though a later study [17] shows a sharper variation for small θ. Clearly, more studies are required to understand the variation of γ close to the apex.

The importance of the apex and its immediate neighbourhood arises from the fact that for sharp emitters, the enhancement factor generally falls rapidly away from the apex even for a decrease in height by only R_a. As a result, the tunneling transmission coefficient can fall by several orders of magnitude rendering the rest of the emitter inconsequential. The emitter current can thus be expressed as

$$I = \int_0^{\rho_{max}} 2\pi\rho\sqrt{1 + (dz/d\rho)^2}J(\mathbf{r})\ d\rho \tag{1}$$

where $\mathbf{r} = (\rho, z)$ is a point on the emitter surface, $\rho = \sqrt{x^2 + y^2}$ and $\rho_{\max}$ is a cutoff set by accuracy requirements. Here $J(\mathbf{r})$ is the local current density [18–23] on the emitter surface, calculated by taking into account the local field enhancement factor $\gamma(\mathbf{r})$. The enhancement factor $\gamma(\mathbf{r})$ around the apex thus holds the key in any field emission calculation.

In the following, we shall first study the field enhancement factor for the hemiellipsoid and cast it in a generalized form $\gamma = \gamma_a \cos\tilde{\theta}$ where $\tilde{\theta}$ is defined using normalized co-ordinates $(\tilde{\rho}, \tilde{z})$. We shall then deal with locally quadratic emitter tips and show numerically that the enhancement factor variation is well described by this generalization.

It may be noted that for all the cases considered in this paper, the emitting tip is at lower electrostatic potential compared to the anode plate so that electric field is directed towards the tip surface. Thus the force on an electron near the emitting surface is directed away from emitter surface. Unless otherwise stated, symbol $\boldsymbol{E}$ is

* Corresponding author.
E-mail address: dbiswas@barc.gov.in (D. Biswas).

https://doi.org/10.1016/j.ultramic.2017.10.016
0304-3991/© 2017 Elsevier B.V. All rights reserved.

used to denote the classical electrostatic field, the symbol E is used to denote its signed magnitude and classical electrostatic potential is denoted by V, with $\boldsymbol{E} = -\nabla V$ [24].

2. Field enhancement for the hemiellipsoid

The vertical hemiellipsoid on a grounded conducting plane placed in an external electrostatic field $(-|E_0|\hat{\boldsymbol{z}})$ pointing along the axial direction is one of few analytically solvable models that have helped in understanding local field enhancement. It is convenient to work in *prolate spheroidal coordinate* system (ξ, η, ϕ) [25]. These are related to the Cartesian coordinates by the following relations:

$$x = L\sqrt{(\eta^2-1)(1-\xi^2)}\cos\phi$$
$$y = L\sqrt{(\eta^2-1)(1-\xi^2)}\sin\phi$$
$$z = L\xi\eta, \tag{2}$$

where $L = \sqrt{h^2 - b^2}$, h is the height and b is the radius of the base of the hemiellipsoid respectively. Note that a surface obtained by fixing $\eta = \eta_0$ in this coordinate system is an ellipsoid. For a prolate hemiellipsoid on a grounded plane in the presence of an external electrostatic field $-|E_0|\hat{\boldsymbol{z}}$, the solution of Laplace equation may be written as [9,26],

$$V(\eta, \xi) = \xi\eta\Big[C' + D'\Big(\frac{1}{2}\ln\frac{\eta+1}{\eta-1} - \frac{1}{\eta}\Big)\Big], \tag{3}$$

where $C' = L|E_0|$ and

$$D' = -L|E_0|\Big(\frac{1}{2}\ln\frac{\eta_0+1}{\eta_0-1} - \frac{1}{\eta_0}\Big)^{-1} \tag{4}$$

where $\eta = \eta_0$ is the surface of the emitter.

In order to relate this to the enhancement factor, γ, we need to find the normal derivative of the potential, V at the surface of the emitter. To do so, we first note that

$$\boldsymbol{E}_{local} = -\hat{\boldsymbol{\eta}}\Big[\frac{1}{h_\eta}\frac{\partial V}{\partial \eta}\Big]_{\eta=\eta_0} \tag{5}$$

where

$$h_\eta = \sqrt{\frac{L^2}{\eta_0^2 - 1}(\eta_0^2 - \xi^2)}. \tag{6}$$

The magnitude of the local electric field normal to the surface $\eta = \eta_0$ is thus given by

$$E_{local} = -\frac{\xi}{h_\eta}\Big[C' + \frac{D'}{2}\ln\frac{\eta_0+1}{\eta_0-1} - \frac{D'\eta_0}{\eta_0^2-1}\Big] \tag{7}$$

Note that at the apex of the hemiellipsoid $\xi = 1$. Thus

$$\frac{\gamma}{\gamma_a} = \frac{\xi\sqrt{\eta_0^2-1}}{\sqrt{\eta_0^2-\xi^2}} \tag{8}$$

Further, with $\xi = z/h$, $L^2 = h^2 - b^2$, $R_a = b^2/h$ and $z^2/h^2 + \rho^2/b^2 = 1$, we have

$$\gamma = \gamma_a\xi\sqrt{\frac{b^2}{\frac{b^2}{h^2}z^2 + h^2 - z^2}} \tag{9}$$

so that

$$\gamma = \gamma_a\xi\sqrt{\frac{b^2}{\frac{b^2}{h^2}z^2 + \frac{h^2}{b^2}\rho^2}} \tag{10}$$

and finally

$$\gamma = \gamma_a\frac{z/h}{\sqrt{(z/h)^2 + (\rho/R_a)^2}}. \tag{11}$$

With $\tilde{z} = z/h$ and $\tilde{\rho} = \rho/R_a$, we define

$$\cos\tilde{\theta} = \frac{\tilde{z}}{\sqrt{\tilde{z}^2 + \tilde{\rho}^2}} \tag{12}$$

so that $\gamma = \gamma_a\cos\tilde{\theta}$. In the limit of the hemisphere where $h = R = R_a$, $\tilde{\theta} = \theta$. Thus, both the hemiellipsoid and hemisphere can be described by Eq. (11).

3. Quadratic surfaces

Generic smooth axially symmetric vertical emitter tips can be described as $z = z(\rho)$. A Taylor expansion at the apex yields

$$z = h + \frac{1}{2}\Big(\frac{d^2z}{d\rho^2}\Big)_{\rho=0}\rho^2 + \dots \tag{13}$$

$$\simeq h\Big[1 - \frac{1}{2}\frac{\rho}{R_a}\frac{\rho}{h}\Big] \tag{14}$$

where R_a is the magnitude of the apex radius of curvature and h is the height of the emitter. We have assumed that the quadratic term is non-zero since $(d^2z/d\rho^2)_{\rho=0} = 0$ implies that the tip is flat rather than having a small radius of curvature characteristic of field emitters. Also, since field emission occurs close to the tip, we shall ignore higher order terms in ρ as in Eq. (14).

The ellipsoid for instance can be expanded as

$$z = h\Big[1 - \frac{1}{2}\frac{\rho}{R_a}\frac{\rho}{h} - \frac{1}{8}\Big(\frac{\rho}{R_a}\Big)^2\Big(\frac{\rho}{h}\Big)^2 - \frac{1}{16}\Big(\frac{\rho}{R_a}\Big)^3\Big(\frac{\rho}{h}\Big)^3 - \dots\Big] \tag{15}$$

which reduces to

$$z = R\Big[1 - \frac{1}{2}\Big(\frac{\rho}{R}\Big)^2 - \frac{1}{8}\Big(\frac{\rho}{R}\Big)^4 - \frac{1}{16}\Big(\frac{\rho}{R}\Big)^6 - \dots\Big] \tag{16}$$

for the sphere. For hemiellipsoidal emitters with large h, a quadratic truncation seems adequate.

Such quadratic emitter tips can thus be considered generic for purposes of field emission. They may be mounted on a variety of bases, ranging from the classical cylindrical post typical of carbon nanotubes to the conical bases of a Spindt array [16] or even be part of compound structures. We shall study the applicability of Eq. (11) for such emitter tips numerically [27,28].

4. Numerical studies

We shall adopt the *nonlinear* line charge model [26,29] to determine the electrostatic potential and thus the field enhancement factor. It consists of a vertically placed line charge of height L on a grounded plane in the presence of an external electrostatic field $|E_0|$. The line charge together with its image and the external field produces a zero-potential surface that coincides with the emitter surface under study. The shape of the zero-potential surface crucially depends on the line charge density. Thus for a linear line charge density the shapes generated are hemiellipsoidal, while non-linear line charge densities can generate a wide variety of shapes including a conical base with a quadratic top.

For our purposes, we consider a polynomial line charge density $\Lambda(z) = \sum_{n=0}^{N} c_n z^n$ with the coefficients c_n chosen appropri-