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Interaction between an edge dislocation and a circular inclusion with an inhomogeneously imperfect interface

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Abstract

This research presents an analytical study of the interaction problem of an edge dislocation with a circular inclusion with a circumferentially inhomogeneously imperfect interface. The interface, which is modeled as a spring (interphase) layer with vanishing thickness, is characterized by that in which there is a displacement jump across the interface in the same direction as the corresponding tractions, and the same degree of imperfection is realized in both the normal and tangential directions. Furthermore, the interface parameter is nonuniform along the interface. In order to arrive at an elementary form solution, we introduce a conformal mapping function. Then the stress field as well as the Peach–Koehler force acting on the edge dislocation can be obtained from the derived complex potentials. Calculations demonstrate that the nonuniform interface parameter has a significant influence on the stress field.

Keywords: Inhomogeneously imperfect interface; Edge dislocation; Circular inclusion; Complex variable method; Conformal mapping

1. Introduction

The interaction problem between (edge or screw) dislocations and elastic inclusions with imperfect bonding at the inclusion–matrix interface (imperfect interface) is an intriguing problem, and has received many investigators' attention (see for example, Wang and Shen, 2002; Fan and Wang, 2003; Sudak, 2003). Here the model of an imperfect interface is based on the assumption that tractions are continuous but displacements are discontinuous across the interface. Furthermore, displacement jumps are proportional, in terms of the 'spring–factor-type' interface parameters, to their respective traction components. The interface

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parameters may be nonuniform along the interface, while uniform interface parameters are adopted in the above listed studies.

In this research, we investigate the interaction between an edge dislocation and a circular elastic inclusion with a circumferentially inhomogeneously imperfect interface. The imperfect interface is characterized by that in which there is a displacement jump across the interface in the same direction as the corresponding tractions, and the same degree of imperfection is realized in both the normal and tangential directions. Furthermore, the interface parameter is nonuniform along the interface and only the simplest kind of inhomogeneously imperfect interface is addressed in this investigation. In order to arrive at an elementary form solution in which all the coefficients are given explicitly (There is no need to solve a coupled infinite system of algebraic equations to arrive at the unknown coefficients), we introduce a special kind of conformal mapping function. Then the stress field as well as the Peach–Koehler force acting on the edge dislocation can be obtained from the derived complex potentials.

2. Basic formulation

As shown in Fig. 1, consider a circular inclusion S_1 of radius R embedded in an unbounded matrix S_2. Throughout the paper, the subscripts 1 and 2 will refer to the regions S_1 and S_2. A straight edge dislocation with Burgers vector $\boldsymbol{b} = \hat{b}_x + \mathrm{i}\hat{b}_y$ is located at $z = z_0$ in the matrix. Since the fact that the edge dislocation can only induce in-plane deformation state, the stresses and the corresponding boundary conditions can be expressed in terms of the two well known Muskhelishvili's complex potentials $\phi(z)$ and $\psi(z)$ as (Muskhelishvili, 1953)

$$\sigma_{rr} + \sigma_{\theta\theta} = 2\left[\phi'(z) + \overline{\phi'(z)}\right],$$

$$\sigma_{rr} - \mathrm{i}\sigma_{r\theta} = \phi'(z) + \overline{\phi'(z)} - \mathrm{e}^{2\mathrm{i}\theta}[\bar{z}\phi''(z) + \psi'(z)], \tag{1}$$

$$p_x + \mathrm{i}p_y = (-\mathrm{i})\left[\phi(z) + z\overline{\phi'(z)} + \overline{\psi(z)}\right]_A^B, \tag{2}$$

$$2\mu(u_r + \mathrm{i}u_\theta) = \mathrm{e}^{-\mathrm{i}\theta}\left[\kappa\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)}\right], \tag{3}$$

where $z = x + \mathrm{i}y = r\mathrm{e}^{\mathrm{i}\theta}$ is the complex coordinate, p_x and p_y are the resultant force components along the x and y directions acting on the left of an arbitrary arc AB in the elastic body. Also, $\kappa = (3 - 4\nu)$ for plane strain, which is assumed in this investigation, and $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress, and μ and ν, where $\mu > 0$ and $0 < \nu \leqslant 0.5$, are the shear modulus and Poisson's ratio, respectively.

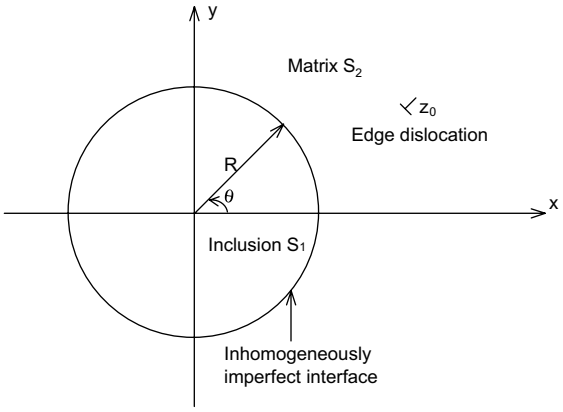

Fig. 1. An edge dislocation interacting with a circular inclusion with an inhomogeneously imperfect interface.