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Iterative reconstruction of magnetic induction using Lorentz transmission electron tomography

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ABSTRACT

Intense ongoing research on complex nanomagnetic structures requires a fundamental understanding of the 3D magnetization and the stray fields around the nano-objects. 3D visualization of such fields offers the best way to achieve this. Lorentz transmission electron microscopy provides a suitable combination of high resolution and ability to quantitatively visualize the magnetization vectors using phase retrieval methods. In this paper, we present a formalism to represent the magnetic phase shift of electrons as a Radon transform of the magnetic induction of the sample. Using this formalism, we then present the application of common tomographic methods particularly the iterative methods, to reconstruct the 3D components of the vector field. We present an analysis of the effect of missing wedge and the limited angular sampling as well as reconstruction of complex 3D magnetization in a nanowire using simulations.

1. Introduction

There is a growing interest in nanomagnetic structures that exhibit complex topological spin states. The research is both fundamental to understand the underlying spin textures and the physical phenomena and applied to ensure the stability of the state and ability to control it. For example, skyrmions exhibit three-dimensional (3D) variation of spin texture in chiral magnetic thin films that are stabilized by the Dzyaloshinskii–Moriya interaction [1,2]. They are often present at specific points in arrays in an otherwise uniform magnetization pattern. They are topologically protected so that they cannot unwind or alter their magnetization continuously. As a result, they are being studied intensively for next generation information storage [3]. Towards achieving this understanding, there is a need to quantitatively map and visualize the entire 3D magnetization in such materials. Not only is 3D information necessary but also the ability to map the magnetization at a high spatial resolution. Lorentz transmission electron microscopy (LTEM) provides an ideal combination of high spatial resolution and ability to quantitatively map the magnetization in nanostructures [4,5].

It has been shown experimentally that combining LTEM with tomographic methods, it is possible to reconstruct the 3D magnetic vector potential of a thin Permalloy (NiFe) islands [6]. The approach used acquisition of four tilt series about two mutually orthogonal axes in the plane of the sample. For each axis, two tilt series were acquired with the sample in as-is orientation and flipped upside down in order to separate the magnetic component of the phase shift from the electrostatic component. The magnetic phase shift tilt series data was then used for reconstructing the individual vector components using the vectorial filtered backprojection algorithm [7,8]. Although the reconstruction showed a correct representation of the spatial variation of the vector components, it suffered from the usual artifacts of limited tilt tomography, such as blurring, and streaking [9]. Additionally the resolution of the components along the z direction is not the same as that in the x and y directions.

In other tomographic techniques such as X-ray computed tomography, the weighted (or filtered) backprojection method is typically not used or only used as a starting point for the 3D reconstruction. The initial estimate obtained using this method is then improved using various forms of iterative reconstruction methods such as algebraic reconstruction technique (ART) or simultaneous iterative reconstruction technique (SIRT) [10]. These methods rely on posing the reconstruction as an inverse problem and rely on solving a set of equations using linear algebra methods. In order to be able to do so, a projector matrix needs to be established that connects the projection data with the data to be reconstructed.

In this paper, we present a formalism to represent the magnetic phase shift data as a Radon transform of the magnetic induction components directly such that we are able to define a projector

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matrix. We will then introduce briefly the four basic reconstruction methods: (1) Weighted Backprojection Method (WBP), (2) Regridding reconstruction method (Gridrec), (3) ART, and (4) SIRT. These methods are commonly available in freeware and commercial softwares and are used for standard scalar tomography [11,12]. These methods are then applied to the reconstruction of vector field components of a uniformly magnetized nanosphere since its 3D magnetic field is analytically known. We perform a detailed analysis of the effect of missing wedge, effect of limited angular sampling on the reconstructed components using the four methods. Finally we show the application of these methods to reconstruct the complex 3D magnetization in a magnetic nanowire with circular cross-section.

2. Methods

2.1. Projection equations for 3D vector field reconstruction

In order to perform 3D reconstruction of the vector field components using various algorithms including iterative methods, it is essential to establish the correct projection equations i.e. forward projection and backward projection for the magnetic phase shift of electrons in terms of the components of magnetic induction of the sample. The magnetic phase shift of the electrons in the presence of a vector potential **A** of the sample can be written from the Aharonov–Bohm relation as [13]

$$\phi_m(\mathbf{r}_\perp) = -\frac{e}{\hbar}\int \mathbf{A}(\mathbf{r})\cdot \mathrm{d}\mathbf{l}; \tag{1}$$

where $\mathbf{r}_\perp$ represents the 2D position vector in the projection plane, which is perpendicular to the direction of projection given by $\mathbf{l}$, and $\mathbf{A}(\mathbf{r})$ represents the vector potential. Considering the direction of projection to be along the z-axis, we have $\mathrm{d}\mathbf{l} = \mathrm{d}z\,\hat{\mathbf{e}}_\mathbf{z}$. Substituting this into the above equation and simplifying, we get

$$\phi_m(\mathbf{r}_\perp) = -\frac{e}{\hbar}\int A_z(\mathbf{r})\,\mathrm{d}z; \tag{2}$$

Taking the derivative with respect to y, we get

$$\frac{\partial \phi_m}{\partial y} = -\frac{e}{\hbar}\int \frac{\partial A_z}{\partial y}\,\mathrm{d}z; \tag{3}$$

where y is defined in the projection plane as $\mathbf{r}_\perp = x\hat{\mathbf{e}}_\mathbf{x} + y\hat{\mathbf{e}}_\mathbf{y}$. From the equation of curl of $\mathbf{A}$, considering only the x component of the curl, we have

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \tag{4}$$

Using Eqs. (3) and (4), we get

$$\frac{\partial \phi_m}{\partial y} = -\frac{e}{\hbar}\int\left(B_x + \frac{\partial A_y}{\partial z}\right)\mathrm{d}z; = -\frac{e}{\hbar}\left(\int B_x\,\mathrm{d}z - \int \frac{\partial A_y}{\partial z}\,\mathrm{d}z\right); \tag{5}$$

The second term of the above equation becomes zero, since the vector potential is assumed to be zero at infinity. Hence the equation relating the magnetic phase shift with the magnetic induction can be written as

$$\frac{\partial}{\partial y}\phi_m(x, y) = -\left(\frac{e}{\hbar}\right)\int_{-\infty}^{\infty} B_x(x, y, z)\,\mathrm{d}z; \tag{6}$$

Similar equation for the x derivative of the magnetic phase shift can be derived as

$$\frac{\partial}{\partial x}\phi_m(x, y) = \left(\frac{e}{\hbar}\right)\int_{-\infty}^{\infty} B_y(x, y, z)\,\mathrm{d}z; \tag{7}$$

The above equations are similar to the projection equations of a scalar quantity. In fact they are the real space representation of the equations derived previously by Phatak et al. [8]. Considering a tilt series around the x-axis, for a given value of x, in the y–z plane, the vector component B_x remains invariant as a function of the tilting of the sample by θ or equivalently tilting the beam by $-\theta$. Thus the projection along any line in the (y,z) plane can be parameterized using the relation $(y(\rho,\xi), z(\rho,\xi)) = ((\rho\cos\theta + \xi\sin\theta), (\rho\sin\theta - \xi\cos\theta))$. Here ρ gives the distance of the line from the origin. This is shown schematically in Fig. 1. Eq. (6) can now be rewritten for any tilt angle θ about the x-axis as

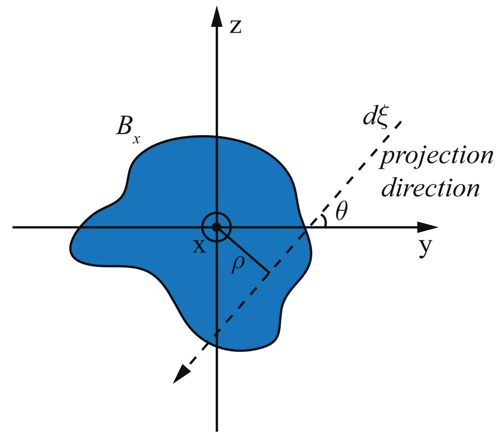

Fig. 1. Schematic figure showing the orientation of projection geometry. The figure shows the y–z plane at $x=0$ and the projection direction dξ at an angle of θ and a distance ρ from the origin.

$$\frac{\partial}{\partial y}\phi_m(x, \rho, \theta) = -\left(\frac{e}{\hbar}\right)\int_{-\infty}^{\infty} B_x(x, \rho\cos\theta + \xi\sin\theta, \rho\sin\theta - \xi\cos\theta)\,\mathrm{d}\xi; \tag{8}$$

If we compare this equation with the definition of Radon transform:

$$R(\theta, \rho) = \int_{-\infty}^{\infty} A(\rho\cos\theta - s\sin\theta, \rho\sin\theta + s\cos\theta)\,\mathrm{d}s, \tag{9}$$

we can see that the Radon transform of a 2D slice of B_x in (y,z) plane for a given value of x gives the respective sinogram at that x in the projection array. The projections thus obtained are essentially the derivative of the magnetic phase shift with respect to y. This establishes the forward projection equation that can be used for computing the projections from the vector field component. The backward projection equations can then be simply computed using the inverse Radon transform as

$$B_x(x, y, z) = \int_0^{\pi} \nabla_y \phi_m(x, y\cos\theta + z\sin\theta, \theta)\,\mathrm{d}\theta. \tag{10}$$

From henceforth, the gradient of the phase shift data is referred to as the projection data. Similar equation for the y component of the magnetic induction can be derived using Eq. (7) and relating it to the tilt series about the y-axis as follows:

$$B_y(x, y, z) = \int_0^{\pi} \nabla_x \phi_m(x\cos\gamma + z\sin\gamma, y, \gamma)\,\mathrm{d}\gamma, \tag{11}$$

where γ is the tilt angle about the y-axis. The third component of the magnetic induction, B_z, can then be computed using the zero-divergence condition, $\nabla\cdot\mathbf{B} = 0$, as

$$B_z(x, y, z) = -\int\left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y}\right)\mathrm{d}z \tag{12}$$

These relations can now be used in various reconstruction algorithms including iterative methods to reconstruct the 3D