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Three-wave electron vortex lattices for measuring nanofields

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ABSTRACT

It is demonstrated how an electron-optical arrangement consisting of two electron biprisms can be used to generate three-wave vortex lattices with effective lattice spacings between 0.1 and 1 nm. The presence of vortices in these lattices was verified by using a third biprism to perform direct phase measurements via off-axis electron holography. The use of three-wave lattices for nanoscale electromagnetic field measurements via vortex interferometry is discussed, including the accuracy of vortex position measurements and the interpretation of three-wave vortex lattices in the presence of partial spatial coherence.

1. Introduction

Apart from their intrinsic appeal from a fundamental perspective, vortical electron wavefields may prove very useful for the measurement of nanoscale electromagnetic fields. A number of methods have now been demonstrated for producing vortical electron wavefields, ranging from holographic masks [1] to optical aberrations [2,3]. In addition to isolated vortices, methods for producing both light and electron wavefields containing arrays of vortices have also been pursued. One possible application of electron vortex arrays is to use the vortices as fiducial markers to measure an electromagnetic field via its effect on the phase of the wavefield. Such approaches, often termed "vortex interferometry", have been proposed [4] and applied [5,6] in light optics. Vortex interferometry permits a real-space approach to phase retrieval [6], and compared to most Fourier-space reconstruction schemes, has advantages in that it enables a direct measurement of the unwrapped phase, as well as potentially enabling better spatial resolution by bypassing the requirement of sideband separation.

A light or an electron vortex consists of a point at which the amplitude of the wavefield is zero, and around which the phase of the wavefield winds by some (non-zero) integer multiple of 2π. The number of 2π windings, taking into account the sign of the winding as an anticlockwise path is traversed, determines the topological charge of the vortex. A vortex lattice is a periodic array of such vortices. The phenomenon of vortex lattice generation from the interference of three, or more, coherent, non-coplanar plane waves has been studied extensively [7–11]. In light optics, plane waves possessing the required coherence are usually generated experimentally by amplitude division of laser light in an interferometer [6,8,12,13], though vortex lattice generation by wavefront division has also been analyzed [14] and demonstrated experimentally [15].

In the case of electrons, several methods are available for generating multiple interfering plane waves and hence vortex lattices. Perhaps the most obvious of these is amplitude division via diffraction from a crystal, where vortices can be generated when the diffracted beams interfere in the image plane [16]. However, to be useful for electromagnetic field measurements, the generation and interference of the plane waves must be performed in a highly controllable and repeatable way. This is unlikely to be achieved using the method just stated. On the other hand, it *can* be achieved by wavefront division of the beam using multiple electron biprisms [17,18]. Such an approach has been demonstrated recently, where two orthogonal biprisms were used to generate four-wave interference patterns containing electron vortices [19]. In fact, similar four-wave interference experiments were demonstrated and discussed in detail much earlier [18], though not specifically in relation to vortices.

For four interfering waves, however, the existence of a vortex at a point of zero intensity is dependent on the relative phases of the waves, and for certain relative phases non-vortical intensity zeros are obtained instead [9,19]. Moreover, four-wave vortices can follow rather complicated paths in three-dimensional space, and they can exhibit vortex creation and annihilation [9], meaning that the vortices can be present at some positions along the optic axis, and not others. However, most importantly, four-wave vortices are unstable with respect to a perturbation in the phase of one of the waves, an aspect that was also emphasized in the work of Eastwood

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et al. [6]. Hence, the use of four interfering waves is ill-suited to vortex interferometry.

Three-wave vortex lattices, on the other hand, are considerably simpler, though to the best of the authors' knowledge, this is the first work to consider them for electrons. An intensity zero produced by three interfering plane waves is guaranteed to contain a vortex. Under free-space propagation, three-wave vortices trace out straight parallel lines, and they do not exhibit creation or annihilation [7,9]. They are also stable with respect to a phase perturbation in one of the waves [6]. A three-wave vortex lattice is also robust with respect to coherent lens aberrations, which merely cause a rigid displacement of the lattice. Hence the conditions for three-wave vortices are considerably more relaxed, potentially enabling them to be used more effectively for electromagnetic field measurements via electron vortex interferometry.

The present work demonstrates the generation and phase measurement of three-wave electron vortex lattices using a transmission electron microscope (TEM) equipped with three electron biprisms. Two biprisms are used to create a three-wave lattice. However, as with all vortex lattices, the interference patterns, taken alone, contain an ambiguity: there exist two lattice configurations, of opposite topological charges, that have identical interference patterns [20]. To prove unequivocally the existence of vortices, and to resolve the topological charge ambiguity, a phase measurement scheme is required. In this work, a third biprism is therefore used to perform a phase measurement of the vortex lattice, including the topological charge, by means of off-axis electron holography. It is shown that this approach enables a relatively simple generation and measurement scheme for three-wave electron vortex lattices.

2. Theoretical background

This section briefly reviews the theory of three-wave vortex lattices. Consider the electron wave function resulting from the coherent superposition of three, non-coplanar plane waves, which for the moment are assumed to be of unit amplitude:

$$\psi(x) = e^{2\pi i k_a \cdot x} + e^{2\pi i k_b \cdot x} + e^{2\pi i k_c \cdot x}, \tag{1}$$

where x is a three-dimensional position vector, and k_a, k_b, k_c are the wave vectors. A coordinate system is chosen such that the z axis, the optic axis, is normal to the plane defined by the tips of the vectors k_a, k_b, k_c. In this coordinate system, the three wave vectors have a common z component k_z, and the wave function can be written in the form

$$\psi(\mathbf{x}, z) = e^{2\pi i k_z z}(e^{2\pi i \mathbf{k}_a \cdot \mathbf{x}} + e^{2\pi i \mathbf{k}_b \cdot \mathbf{x}} + e^{2\pi i \mathbf{k}_c \cdot \mathbf{x}}), \tag{2}$$

where bold symbols denote two-dimensional vectors lying transverse to the optic axis. Factoring out one of the plane waves, plane wave C, say, the wave function can be written in the form

$$\psi(\mathbf{x}, z) = e^{2\pi i \mathbf{k}_c \cdot \mathbf{x}} e^{2\pi i k_z z}(1 + e^{2\pi i \mathbf{k}_{ac} \cdot \mathbf{x}} + e^{2\pi i \mathbf{k}_{bc} \cdot \mathbf{x}}), \tag{3}$$

where $\mathbf{k}_{ac} = \mathbf{k}_a - \mathbf{k}_c$ and $\mathbf{k}_{bc} = \mathbf{k}_b - \mathbf{k}_c$.

The existence of a vortex requires that the absolute value of the wave function vanishes at the vortex position (since the phase at that position is undefined). This demands that the three terms contained in the parentheses in Eq. (3) sum to zero. Pictorially, if each of these three terms is drawn as a phasor in the complex plane, the phasors must form a closed triangle. A closed triangle can actually be formed in two distinct ways, giving rise to vortices and antivortices, respectively, as shown schematically in Fig. 1a. Hence vortices and antivortices correspond to the phase conditions

$$\mathbf{k}_{ac} \cdot \mathbf{x}_{mn}^{\pm} = m \pm \tfrac{1}{3}, \quad \mathbf{k}_{bc} \cdot \mathbf{x}_{mn}^{\pm} = n \mp \tfrac{1}{3}, \tag{4}$$

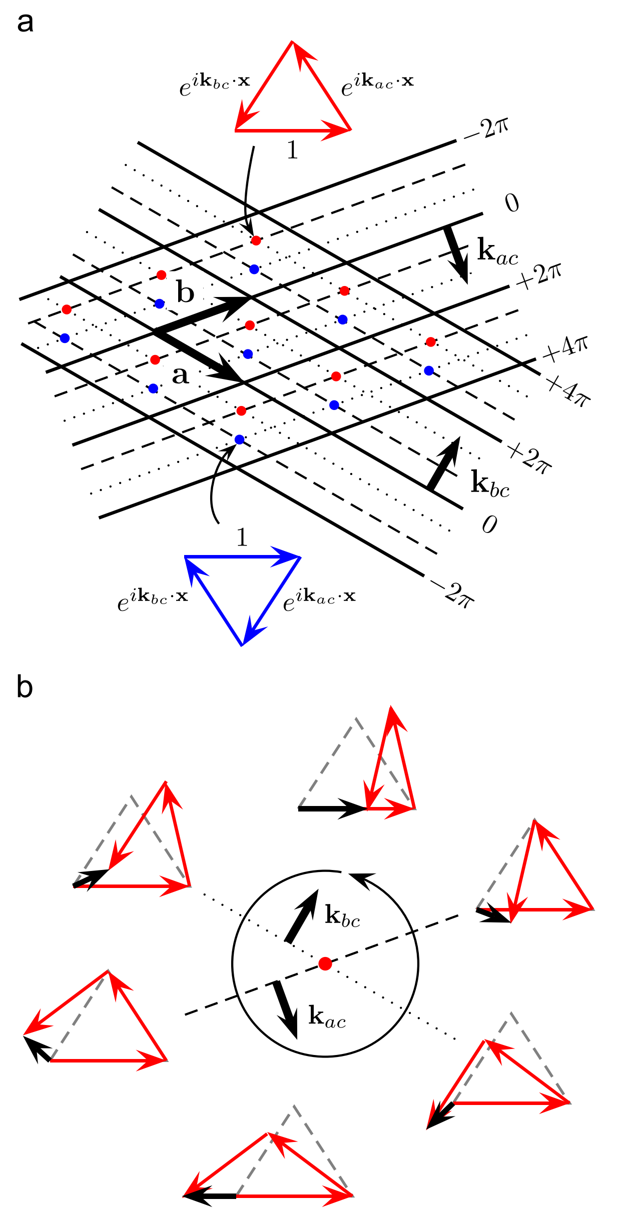

Fig. 1. Schematic representation of a three-wave vortex lattice. (a) The wave function in a plane perpendicular to the optic axis (as defined in the text). Vortices and antivortices (red and blue dots, respectively) exist wherever the three phasors form a closed triangle in the complex plane. The lattice vectors **a** and **b** are the duals of the transverse wave vectors $\mathbf{k}_{ac}$ and $\mathbf{k}_{bc}$, respectively. (b) The evolution of the phasor sum as an anticlockwise path is traversed around a vortex. The phasor sum (black arrow) is seen to acquire a phase of $+2\pi$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

where m and n are integers, and the upper and lower signs apply to vortices and antivortices, respectively. By defining the vectors **a** and **b** to be the duals of $\mathbf{k}_{ac}$ and $\mathbf{k}_{bc}$, the vortex and antivortex positions are given by $\mathbf{x}_{mn}^{\pm} = (m \pm 1/3)\mathbf{a} + (n \mp 1/3)\mathbf{b}$, where the different values of m and n give rise to a periodic vortex lattice in two dimensions. It is noted the vortex positions are independent of z. Hence, in three-dimensional space each vortex will trace out a line parallel to the optic axis as it was defined above.

The fact that, under the stated conditions, a point of vanishing absolute value *must* contain a vortex (or antivortex) is demonstrated pictorially in Fig. 1b. There we see that the resultant of the three-phasor sum necessarily acquires a phase of $+2\pi$ (or -2π for an antivortex) as an anticlockwise path is traversed in the immediate neighborhood of the point in question. Note that the