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Two-dimensional matrix-based non-complex axis-of-rotation error modeling

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ABSTRACT

This work applies to the evaluation of a spindle as an integral part of a machine whose task is the measurement or modifying the shape or other parameter of a workpiece. This includes the co-influences of the spindle, its drive units and structural loop of the machine. First, the matrix-based two-dimensional simplified solution free of complex numbers is described. Second, bounds that encompass all possible radial error motions are determined from the matrix-based model. Third, it is shown that excited unidirectional resonances are improperly perceived as retrograde errors during evaluation due to the calculation and removal of reference ball eccentricity effects. Fourth, using a Fourier multi-dimensional spatial spectrum model, the stator referenced spindle error can result in a particular mth spectral component producing an effect at both the $m+1$ and $m-1$ spatial frequencies on the workpiece if the tool is mounted on the rotor. Fifth, it is shown how to directly calculate the (+ and −) mth characteristic rotating vectors. Sixth, a quick consideration will be given to estimating 3D part errors on radially varying surfaces such as but not limited to cones and spheres.

1. Introduction

An axis-of-rotation in its simplest form is embodied by a spindle consisting of a rotor, a stator, and a bearing element between the two which provides one degree of freedom rotationally while nominally constraining the other five. Applications spanning many dimensional orders from micro-mechanisms to wind turbines have associated axes-of-rotation. In manufacturing, a spindle can be used to hold either a workpiece or a tool/probe. An axis-of-rotation is generated when a spindle's rotor turns with respect to its stator. Evaluating this relative motion between the rotor and the stator has developed through the years [1–4] and is embodied in the ASME B89.3.4 [5] and ISO 230-7 [6] standards which enable buyers and sellers to agree upon specifications. A part of such an evaluation is a *synchronous error motion polar plot* of the rotationally synchronous radial deviations of the spindle error in the *rotating or fixed sensitive direction* (considered radial in the B89.3.4 standard). An example of plots for the *rotating sensitive direction* is shown in Fig. 1 for a spindle on a machine operating at two different speeds.

Often a single *total error motion polar plot* is used for graphically showing the total spindle error; however, the form errors (usually radial) produced in a part by the spindle's errors will be different for each application (mill-boring, turning etc.) as well as the tool/probe mounting angle. As an example of the differences in effect that can occur for a particular spindle, Fig. 2 shows plots for two calculations of a range of possible tool tip deviations from a pure circular contour for the oblong data used to obtain Fig. 1. One is for a tool mounted on the rotor with the workpiece on the stator and the other is for a tool mounted on the stator with the workpiece on the rotor. Although a single angular location of the tool is chosen for each illustrated case, in application the location can be at any angle. Also shown are the inner and outer boundaries (bold lines) encompassing all the possible contouring deviations due to the possible tool locations on the spindle. Obtaining these inner and outer boundaries is described later.

Increasingly, spindles are being used simultaneously with multiple probes in various orientations [7]. These factors suggest that the evaluation of the errors of a spindle should be reported in such a way that is omni-directionally and application inclusive and not related to any particular metrology sensor location. Thus, it is beneficial to evaluate and report a spindle's possible errors in a way that is both inclusive

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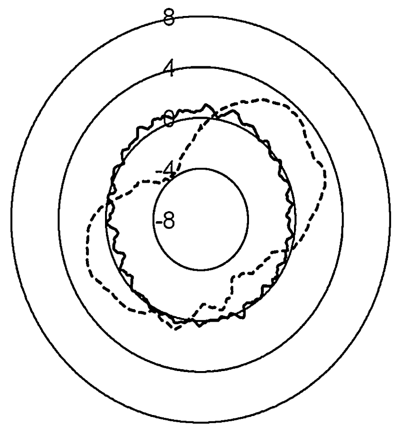

Fig. 1. Polar plot showing the synchronous radial error motion of a spindle. The near circular solid line plot is from data taken at 1000 rpm while the oblong dashed plot is from data of the same spindle taken at 2000 rpm near a resonance mode. The units are in micrometers.

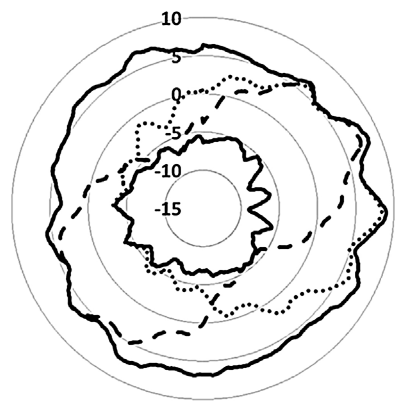

Fig. 2. Two of the possible radial deviations from a pure circular contour of a tool tip in the workpiece reference frame are shown as dashed and dotted from the measured 2000 rpm data of Fig. 1. The solid lines bound all the possibilities of the radial deviations for the spindle. The actual effect depends on the specific tool angle and task performed by the spindle.

and independent of its application for turning, facing, boring, milling, grinding etc while at the same time allowing for the exclusion of task-insensitive errors. Lu [8] showed how to calculate spindle errors and consequences using complex number based calculations. However by calculating the spindle error motion vectors via non-complex 2×2 matrices, two-dimensional motion is readily obtained for general and specific cases. In addition, by inter-related modeling of coordinate frames for the rotor, stator and also workpiece, the contours generated in the rotor frame by a point in the stator frame as the rotor turns and vice-versa are used to show that the workpiece contour deviations are dependent on the angular position of the points as well as the reference frame in which the contours are produced. The case independent bounding model is then presented which encompasses all the possible error cases. Subsequently, the mathematical model is used to demonstrate that unidirectional and retrogressing fundamental order error motions are indistinguishable and perceived the same by common analysis techniques. Theoretical and actual data are used to illustrate the application of the model.

The relationship between errors in the stator coordinate system (milling style) and the errors in the rotor coordinate system (turning style) is addressed utilizing the modeling which incorporates coordinate systems for both the spindle and the rotor. This relationship shows that a mth spatial frequency analysis component in either coordinate system produces an effect at the $m+1$ and $m-1$ spatial frequencies in the other coordinate frame. A method similar to a Fourier transform for two dimensional data is shown including the matrix based calculation using discrete data.

1.1. Physical spindle modeling

Consider a simple spindle consisting of a stator, a rotor, and a bearing. Given a specific spindle state (speed, temperature, etc.), the average line in the stator frame about which any point on the rotor has minimal radial deviations through rotation is the spindle's axis-of-rotation, S_A. The axis line of the rotor, R_A, is the physical line fixed in the rotor frame which has the minimal average distance from S_A as the rotor turns through a large number of revolutions. Let S_O be a predetermined point on S_A beyond the face of the spindle. Let R_O be the point on the rotor which has the minimum separation from S_O as the spindle rotates through a large number of rotations. The stator to rotor lateral shift vector, $\mathbf{\Delta}$. from S_O to R_O varies as θ changes. This is illustrated in Fig. 3 with P being a plane perpendicular to S_A having intersection at S_O.

S_F and R_F are the coordinate system frames associated with the stator and rotor frames, respectively, as illustrated in Fig. 4, having their origins at S_O and R_O, respectively and having their Z axes S_Z and R_Z extending away from the spindle face along S_A and R_A, respectively. S_X and R_X are the respective lines through S_O and R_O which are perpendicular to S_Z and R_Z. Although directionally arbitrary they may