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A precision-to-tolerance ratio model for the assessment of measurements uncertainty

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ABSTRACT

Estimates of process capability indices are distorted by the presence of gauge measurement errors, a matter which results in two quality measures i.e., the actual and observed process capability indices (AC_p and OC_p). Gauge errors (Gauge uncertainty) add distrust to the measure data, as a result, one has to assure the accuracy of the gauge by conducting a gauge repeatability and reproducibility (GR&R) study. In this paper, we present novel relationships between the AC_p and OC_p using the precision-to-tolerance ratio (PTR) to assess the gauge as well as the process capability, simultaneously. Particularly, we will find the chances that an estimated process capability is measured by an erroneous or a perfect gauge. In addition, instead of using the strict threshold values of the PTR to judge the measurement gauge, a novel γ–δ significance characteristic curve will be introduced. The values of γ and δ will describe the accuracy of the measurement system while the critical process capability values/ratios will be computed in the so called χ^2 and C_p domains. The introduced γ–δ curve symbolizes the PTR paradigm, here, the complications associated with the strict PTR thresholds used in the literature to judge the gauge capability will be avoided. The chances of having a measurement under the assumption of AC_p distribution while it is the observed and vice versa will be established as type I and type II errors.

To assess the descriptive merits of the proposed model and guidelines, two case studies from the literature of normally distributed data were addressed. The analysis showed that trustable gauge measurements cannot be proclaimed by just setting strict PTR values.

1. Literature review

Measurement system analysis is crucial for the control and evaluation of the measurement process. As the measurement errors cannot be avoided in measured data, an adequate measurement assessment becomes a necessity. Traditionally, gauge repeatability and reproducibility study which is performed according to the QS9000 standards can address the different variation components in the measurement system to help judge the adequacy of the gauge [1].

For the acceptance standards of a process capability, it is the precision-to-tolerance ratio (PTR) which was considered as a main guideline in the Measurement System Analysis (MSA) manual edited by the three major automobile companies; Ford, GM and Chrysler in the united states [2]. In fact, there exist numerous studies on the settings of PTR and its associated thresholds published by many experts like Tsai [3], Montgomery and Runger [27,28], Levinson [4], Jheng [5], Pearn [22] and Pan [6].

A huge deal of interesting studies were published on process capability indices and their impact on industry, among these we mention Kane [7], Chan et al. [8], Boyles [9], Cheng [10], Johnson [11], Fred [12], Chen et al. [13] and Chen and Chen [14,15]. However, most of such studies addressed the same PTR thresholds set by industry and manufacturers along with some statistical significance and rejection criteria, this also can be found on multivariate quality characteristics [16]. In addition, most of these studies did not consider the gauge errors resulting from the accuracy of measurement instruments, yielding potentially incorrect process capabilities [17,20].

This paper is organized as follows: Section 2 demonstrates an introduction to measurement system analysis. Section 3 presents the critical values and the χ^2 property. The concept of process capability domains is presented in Section 4 followed by the novel PTR model in Section 5. Our γ–δ characteristic curve is illustrated in Section 6 followed by the tradeoffs between the AC_p and OC_p in Section 7. The case studies and validation work is demonstrated in Section 8 followed by the conclusions in Section 9.

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2. Introduction

Process capability indices are powerful measures used to investigate whether the product of a production process conforms to the specification limits. The process capability ratio C_p is calculated as

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \tag{1}$$

the numerator gives the range between the upper and lower specification limits preset by product designers, whereas, the range of the actual process variation is given in the denominator.

Measurement errors are unavoidable; accordingly, the process indices are likely to be distorted. Gauge capability is assessed by different measures, among which is the widely used precision-to-tolerance ratio (PTR) which relates the variability of the gauge to the specifications [18,19].

$$\text{PTR} = \frac{k\hat{\sigma}_g}{\text{USL} - \text{LSL}} \tag{2}$$

where $\hat{\sigma}_g$ is the standard deviation (standard uncertainty) of the gauge, k is the uncertainty coverage factor and $k\hat{\sigma}_g$ is the expanded uncertainty. If PTR < 0.1 then the gauge is capable, whilst if PTR > 0.3 then the gauge is not capable with an indistinct margin left between these limits.

For a measurement system to be deemed satisfactory, the measurement system variability has to be less than a predetermined percentage of the engineering tolerance. The automotive industry action group recommended the above guidelines for acceptance of gauges [1,18,22].

Due to the considerable degree of unreliability, drawing conclusions of whether the process is capable or not cannot be merely indicated by the process capability indices [20]. When gauge errors are found in the measured data, the computed process capability indices will differ. Hence, an observed process capability will definitely be less than the actual process capability due to additional gauge variability.

This study aims at first; declaring if a measurement system is capable or incapable through the newly proposed guidelines and γ–δ characteristic curve, second; judging the process capability by the use of a critical value that takes the measurement errors into account. To achieve these goals, both the observed and the actual process capability indices (OC_p and AC_p) will be formulated as a function of PTR. The tradeoffs between the two indices will be formulated and analyzed. Further, a statistical hypothesis test will be introduced to benchmark the process capability indices with their critical values. Moreover, the distributions of both indices will be addressed to establish the possible error margins (i.e., type I and type II errors). The proposed model and guidelines will diminish the need to state strict PTR thresholds as compared to the currently used guidelines in the literature.

3. The critical values of the actual and observed process capability indices

In measurement system analysis (MSA) gauge repeatability and reproducibility (gauge R&R) study helps to identify the different sources of variability. It identifies two variation components, those are $\hat{\sigma}^2_{\text{repeatability}}$ and $\hat{\sigma}^2_{\text{reproducibility}}$. The square root of the sum of $\hat{\sigma}^2_{\text{repeatability}}$ and $\hat{\sigma}^2_{\text{reproducibility}}$ constitutes the standard uncertainly of the gauge $\hat{\sigma}_g$. The sum of the variability components $\hat{\sigma}^2_g$ and $\hat{\sigma}^2_p$ characterizes the total measurement system variation where $\hat{\sigma}^2_p$ is the part variance [18,21]. That is

$$\hat{\sigma}^2_{\text{total}} = \hat{\sigma}^2_p + \underbrace{\hat{\sigma}^2_{\text{repeatability}} + \hat{\sigma}^2_{\text{reproducibility}}}_{\hat{\sigma}^2_g} \tag{3}$$

Statistical testing can be implemented for further verification the process index C_p, where if $C_p > c$ the process satisfies the quality requirement, while if $C_p \le c$, the null hypothesis cannot be rejected. The quantity c represents a benchmark value of C_p such as 1, 1.33, 1.67, 2, . . ., etc. Thus, using estimates of C_p, a test hypothesis can be set as

$$H_0 : C_P \le c,$$

$$H_1 : C_P > c.$$

Note that the critical values can be found directly from the χ^2 distribution. Since we have a reference parameter σ_{total}, where the estimated value of this parameter is $\hat{\sigma}_{\text{total}} = \sqrt{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2/(n-1)}$, the study of Pan [26] found that:

$$(n-1)\left(\frac{\hat{\sigma}_{\text{total}}}{\sigma_{\text{total}}}\right)^2 \sim \chi^2_{n-1} \tag{4}$$

where the above ratio follows χ^2 distribution. Now since the capability index can be estimated as $\hat{C}_p = (\text{USL} - \text{LSL})/6\hat{\sigma}$, if we divide the capability index C_p by its estimate $\hat{C}_p$, we can obtain the following relationship:

$$\frac{C_p}{\hat{C}_p} = \frac{\hat{\sigma}_{\text{total}}}{\sigma_{\text{total}}} \tag{5}$$

Take the square of this ratio and multiply it by the degrees of freedom $(n-1)$, we get: $(n-1)\left(C_p/\hat{C}_p\right)^2$. This ratio follows χ^2 distribution, with $(n-1)$ degrees of freedom as shown by (4), this is

$$(n-1)\left(\frac{C_p}{\hat{C}_p}\right)^2 \sim \chi^2_{n-1}, \tag{6}$$

In fact this ratio has been verified in so many literature studies [17,26]. Note the critical value can be determined at a significance level of γ in this χ^2 domain, such that:

$$\gamma = P(\hat{C}_p > c_0 | C_p = c) = P\left(\chi^2 < \frac{(n-1)c^2}{c_0^2} | C_p = c\right)$$

Accordingly, we obtain the expression $\left((n-1)c^2/c_0^2\right) = \chi^2_{1-\gamma,n-1}$, where $\chi^2_{1-\gamma,n-1}$ represents the inverse χ^2 value at the lower $(1-\gamma)$ quantile with $(n-1)$ degrees of freedom. Thus the ratio between the critical value and the actual C_p can be represented by

$$\frac{c_0}{C_p} = \sqrt{\frac{n-1}{\chi^2_{1-\gamma,n-1}}} \tag{7}$$

Process capability index is usually assessed by measurement data resulting from a regular gauge which encompasses the part as well as the gauge variances. Consequently, the real observed capability index OC_p is defined as

$$OC_p = \frac{d}{6\hat{\sigma}_{\text{total}}} = \frac{d}{6\sqrt{\hat{\sigma}^2_p + \hat{\sigma}^2_g}} \tag{8}$$

where OC_p is the estimator of the observed index. In this case $\hat{\sigma}_{\text{total}}$ is represented by the summation of the variation due to the part and the gauge, d represents the range between the specification limits (i.e., USL − LSL). Meanwhile, the actual capability index only refers to the part variation, thus another relationship can be stated here as

$$AC_p = \frac{d}{6\sigma_p} \tag{9}$$