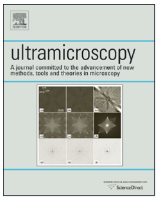
Ultramicroscopy 133 (2013) 67–71

Contents lists available at SciVerse ScienceDirect

Ultramicroscopy

journal homepage: www.elsevier.com/locate/ultramic



Apparent increase in the thickness of superconducting particles at low temperatures measured by electron holography

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ARTICLE INFO

Article history:
Received 11 March 2013
Received in revised form
10 May 2013
Accepted 12 May 2013
Available online 22 May 2013

Keywords:
Electron holography
Thickness measurement
Interference micrograph
Superconductors

ABSTRACT

We predict that superconducting particles will show an apparent increase in thickness at low temperatures when measured by electron holography. This will result not from a real thickness increase, rather from an increase in the mean inner potential sensed by the electron wave traveling through the particle, originating in expansion of the electronic wavefunction of the superconducting electrons and resulting negative charge expulsion from the interior to the surface of the superconductor, giving rise to an increase in the phase shift of the electron wavefront going through the sample relative to the wavefront going through vacuum. The temperature dependence of the observed phase shifts will yield valuable new information on the physics of the superconducting state of metals.

1. Introduction

Electron holography is a sensitive tool to measure the thickness of small particles [1], as demonstrated experimentally already several decades ago. Initially, experiments yielded a resolution of only tens of nanometers [2] corresponding to phase difference of 2π between neighboring contour lines. However it was soon shown that the resolution could be improved by large factors using phase difference amplification techniques, and phase shift differences of $2\pi/100$ could be detected [3]. Interference micrographs as shown in Figs. 1 and 2 depict contours of constant phase shift, and for a spherical sample the contours are circles as shown in those figures. The phase difference between neighboring contours in Figs. 1 and 2 is π (two-times phase difference amplification).

A given circle of radius r in the contour map corresponds to a given phase shift $\Delta\varphi$. Imagine that the thickness of the sample was to increase as the temperature is lowered. The phase shift for that value of r will increase, so the radius of the circle that will give to the original phase shift $\Delta\varphi$ has to increase. In other words, the contours of given phase shift will move *outward* as shown in Figs. 1 and 2 by the dashed lines. In this paper we predict that this will be observed for any superconducting particle cooled sufficiently below its critical temperature.

The contour shift will appear to indicate that the sphere becomes a prolate ellipsoid of revolution with the longer axis along the beam direction. In reality, no change in the physical dimensions of the sample will have occurred. The behavior shown in Figs. 1 and 2 will signal that a change in the mean inner potential sensed by the electron beam will have occurred, originating in electronic charge redistribution inside the superconductor: namely, that negative charge has moved from the interior of the superconductor to the surface. The predicted charge distribution, electric field and electric potential inside the superconductor is shown schematically in Fig. 3. For a spherical particle no electric field outside the particle is generated through this charge redistribution due to the spherical symmetry and the fact that the particle remains charge neutral.

2. Phase shift

In an electron holography experiment, the phase shift of the electron wave going through the particle at a distance r_0 from the center of the sphere, relative to a reference wave going through vacuum is given by [1]

$$\varphi(r) = C_T \int_{-z_0(r_0)}^{z_0(r_0)} [V_0 + V_{ce}(r(z, r_0))] \tag{1}$$

where

$$C_T = \frac{2\pi e}{\lambda T} \frac{T + E_0}{T + 2E_0} \tag{2}$$

with T the kinetic energy of the electron, E_0 the electron rest energy and $\lambda = hc/\sqrt{T^2 + 2TE_0}$ the electron wavelength. The points where the electron wave enters and exits the spherical particle for

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http://dx.doi.org/10.1016/j.ultramic.2013.05.007

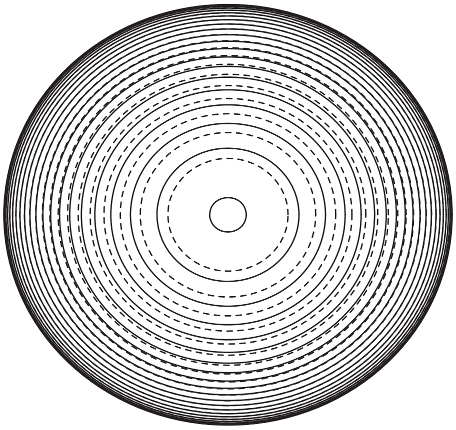

Fig. 1. Predicted interference micrograph of a spherical superconducting particle of radius 500 nm, mean inner potential $V_0 = 10$ V, London penetration depth $\lambda_L = 40$ nm and lower critical magnetic field $H_{c1} = 100$ G with 300 kV electrons, two-times phase amplified. The full lines indicate the contours of constant phase above T_c, the dashed lines at temperatures well below T_c.

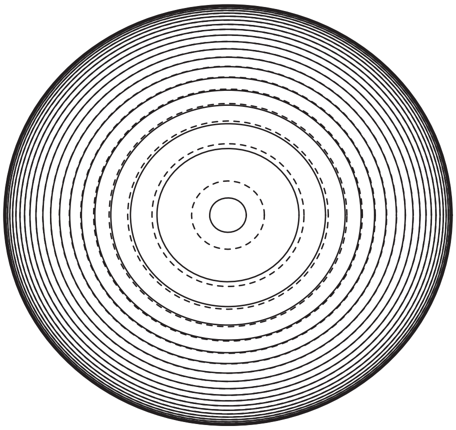

Fig. 2. Same as Fig. 1 for $\lambda_L = 150$ nm and $H_{c1} = 73$ G, parameters appropriate for *YBCO*. Both here and in Fig. 1, each full line contour is predicted to move gradually *outward* to the closest dashed line contour as the temperature is lowered below T_c. Phase amplification factor is 2 as in Fig. 1.

a phase contour of radius r_0 are $\pm z_0(r_0)$, with

$$z_0(r_0) = \sqrt{R^2 - r_0^2} \tag{3}$$

with R the radius of the particle. V_0 is the ordinary mean inner potential [4,5] of the solid which is expected to be constant inside the material. The additional potential V_{ce} resulting from the charge redistribution shown in Fig. 3 is not constant but depends on the radial distance r to the center of the sphere

$$r(z, r_0) = \sqrt{r_0^2 + z^2}. \tag{4}$$

The total potential energy of the electron wave inside the material

$$eV(z) = e(V_0 + V_{ce}(r(z, r_0))) \tag{5}$$

depends on the position along the electron trajectory, as shown in Fig. 4 for several values of r_0. This additional potential well increases the phase shift of the high energy electron wave going through the material, giving rise to the contour shifts shown in Figs. 1 and 2. For the examples of Figs. 1 and 2 we used T=300 keV, R=500 nm.

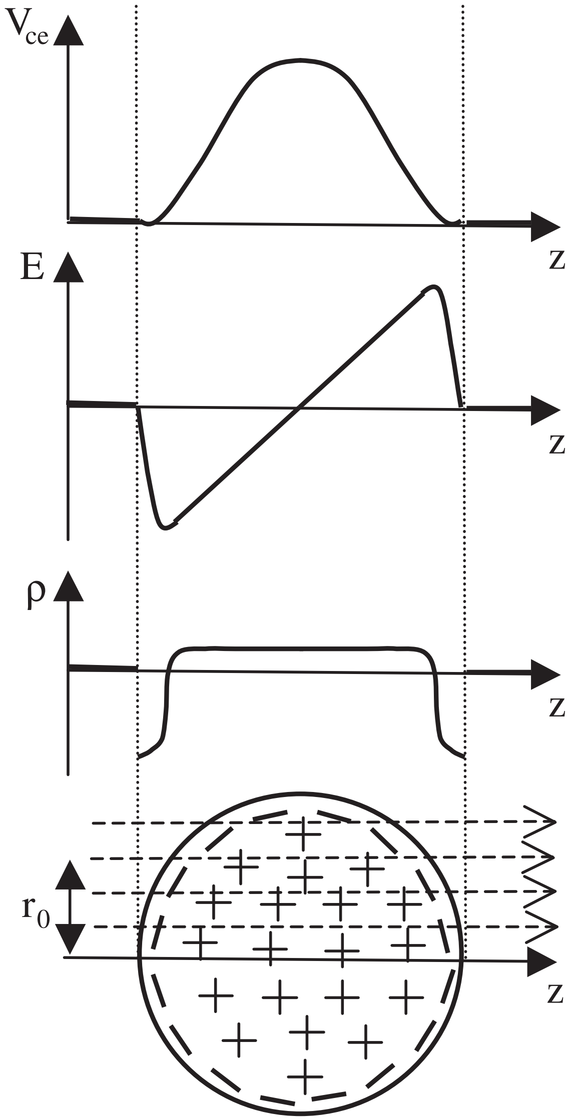

Fig. 3. Predicted charge distribution for a superconducting spherical particle at low temperatures, and resulting charge density ρ, electric field E and electric potential V_{ce} (schematic) sensed by the beam electrons traveling along the sphere diameter. The electric field points radially outward. The charge density ρ becomes negative at a distance λ_L (London penetration depth) from the surface, at that point the electric field magnitude reaches maximum value. The dashed lines show other possible trajectories for the beam electrons, all giving rise to a positive phase shift.

3. Electric potential due to charge expulsion

The theory of hole superconductivity [6] predicts that the charge distribution in any superconductor at sufficiently low temperatures is macroscopically inhomogeneous, with more negative charge near the surface and more positive charge in the interior, resembling a 'giant atom' [7], as shown schematically in Fig. 3. The charge inhomogeneity is very small: the excess negative charge resides within a London penetration depth (typically several hundreds Angstrom) of the surface and is only of order 1 extra electron per 1 million atoms [8].

There is no direct experimental evidence of this physics so far, nor, we argue, is it ruled out by any existing experiment. Electron holography experiments on superconductors performed in the past [9] have not tested this physics. A compelling reason in favor of this scenario is that it provides a *dynamical* explanation of the Meissner effect [10], not provided by the conventional BCS theory: