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Application of the continuous wavelet transform in periodic error compensation

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ABSTRACT

This paper introduces a new discrete time continuous wavelet transform (DTCWT)-based algorithm, which can be implemented in real time to quantify and compensate periodic error for constant and non-constant velocity motion in heterodyne displacement measuring interferometry. It identifies the periodic error by measuring the phase and amplitude information at different orders (the periodic error is modeled as a summation of pure sine signals), reconstructs the periodic error by combining the magnitudes for all orders, and compensates the periodic error by subtracting the reconstructed error from the displacement signal measured by the interferometer. The algorithm is validated by comparing the compensated results with a traditional frequency domain approach for constant velocity motion. The algorithm demonstrates successful reduction of the first order periodic error amplitude from 4 nm to 0.24 nm (a 94% decrease) and a reduction of the second order periodic error from 2.5 nm to 0.3 nm (an 88% decrease). The algorithm also reduces periodic errors for non-constant velocity motion overcoming limitations of existing methods.

1. Introduction

Displacement measuring interferometry provides high resolution and accuracy for dimensional metrology and is used in a number of applications including semiconductor fabrication and linear stage calibration. Heterodyne (two-frequency) Michelson-type are often selected. The interference between the reference and measurement signals is observed at a photodetector, where current is generated proportional to the optical interference signal. The current is processed and converted to voltage and the phase between the reference signal and the signal after displacement is determined by phase-measuring electronics. The measured phase change is nominally linearly proportional to the displacement of the measurement target. However, errors and defects in the optical system components cause frequency mixing between the polarized reference and measurement arms. This frequency mixing causes periodic error to be superimposed on the measured displacement signal. For heterodyne interferometers, both first and second order periodic errors can occur, which correspond to one and two periods per displacement fringe. The periodic error can limit the accuracy of the heterodyne interferometer to the nanometer level (or higher) depending on the optical setup.

Previous research has demonstrated a frequency domain approach to periodic error identification [1–3], where the periodic error is measured by calculating the Fourier transform of the time domain data collected during constant velocity target motion. The periodic errors are then determined from the relative amplitudes of the peaks in the frequency spectrum. For an accelerating or decelerating motion, however, the Doppler frequency varies with velocity. In this case, the frequency domain approach is not well-suited because the Fourier transform assumes stationary signals. To overcome this limitation, a wavelet-based analysis is applied here to measure and compensate periodic error.

A continuous wavelet transform (CWT) of a time domain signal provides information in both the temporal and frequency domains [4]. For example, calculating the Morlet CWT enables the frequency content of a signal to be observed at different times [5–7]. The CWT can be more informative than the Fourier transform because the CWT shows the relationship between frequency content and signal based on the wavelet scale and the time period. This enables the frequency and time information of the signal to be determined simultaneously by applying an appropriate wavelet. When applied to non-stationary signals, the CWT can supply accurate frequency information continuously.

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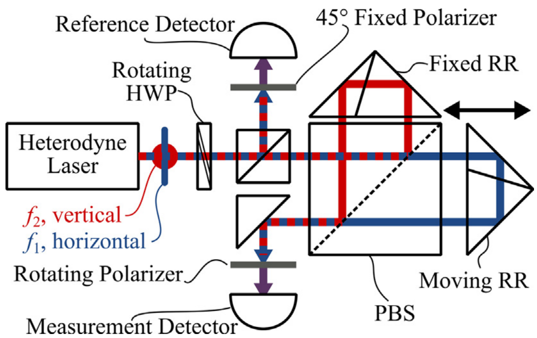

Fig. 1. Schematic of heterodyne interferometer setup. Optical components include: retroreflectors (RR), polarizing beam splitter (PBS), polarizers, half wave plate (HWP), and photodetectors.

In this work, a wavelet-based approach is used to capture both the temporal and spectral components of periodic error to allow for non-constant velocity error compensation. This paper discusses periodic error, the CWT in its general form, and introduces a new wavelet-based approach to measure and compensate periodic error for both constant and non-constant velocity target motions. The effectiveness of the wavelet-based approach to detecting and compensating periodic error is compared to the traditional Fourier-based approach for constant velocity motion. The effectiveness of the wavelet-based approach is also tested for non-constant velocity motion to demonstrate that utilizing wavelets allows for periodic error compensation for time-varying and frequency-varying errors.

2. Background

2.1. Periodic error

Heterodyne Michelson interferometers (Fig. 1) use a two-frequency laser source and separate the two optical frequencies into one fixed length and one variable length path via polarization. Ideally these two beams are linearly polarized and orthogonal so that only one frequency is directed toward each path. An interference signal is obtained by recombining the light from the two paths; this results in a measurement signal at the heterodyne (split) frequency of the laser source. This measurement signal is compared to the optical reference signal. Motion in the measurement arm causes a Doppler shift of the heterodyne frequency which is measured as a continuous phase shift that is proportional to displacement. In practice, due to misalignment of optical components, component imperfections, and elliptical polarization, undesirable frequency mixing occurs which yields periodic errors.

Fedotova [8], Quenelle [9], and Sutton [10] first investigated periodic error in heterodyne Michelson interferometers. Subsequent studies of periodic error and its reduction have been reported in the literature [8–38]. Researchers have analyzed and applied different methods to measure and compensate periodic error, including the frequency domain approach [1–3] and several time domain measure-and-compensate algorithms [39–43].

2.2. Continuous wavelet transform

A wavelet function of time, $\psi(t)$, is a finite energy function with an average of zero and is usually normalized to a unit value [44],

$$\bar{\psi}(t) = \int_{-\infty}^{+\infty} \psi(t)dt = 0, \qquad (2.1)$$

$$\left\|\psi(t)\right\|^2 = \int_{-\infty}^{+\infty} \left|\psi(t)\right| dt = 1. \qquad (2.2)$$

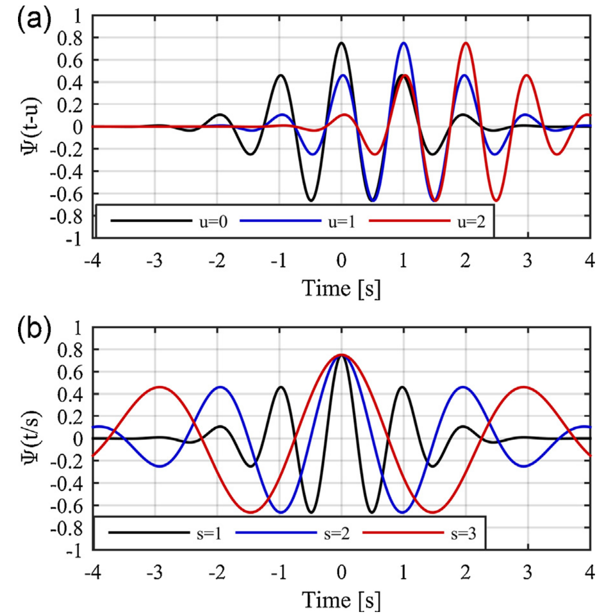

Fig. 2. Translation and dilation of the mother wavelet. (a) Shifting a wavelet in time is translation. (b) Stretching or squeezing a wavelet is dilation.

A wavelet family can be generated from a "mother" wavelet by translating it via the shift parmeter, $u \in \Re$ and dilating the wavelet via the scale parameter, $s > 0$ (Fig. 2). This series of wavelets can be expressed as

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right). \qquad (2.3)$$

In this research, a continuous wavelet transform (CWT) is used to analyze the displacement signal, $x(t)$, which is measured by the heterodyne interferometer, with a wavelet function $\psi(t)$. For this one-dimensional signal, $x(t)$, the CWT result W is defined as the convolution of $x(t)$ with a scaled and translated variation of the mother wavelet $\psi(t)$ via

$$Wx(u,s) = \int_{-\infty}^{+\infty} x(t)\psi^*_{u,s}(t)dt = \int_{-\infty}^{+\infty} x(t)\frac{1}{\sqrt{s}}\psi^*\left(\frac{t-u}{s}\right)dt, \qquad (2.4)$$

where * indicates the complex conjugate. One property of CWT is its linearity,

$$\left(W\sum_{i=1}^{N}\alpha_i x_i\right)(u,s) = \sum_{i=1}^{N}\alpha_i[Wx_i(u,s)], \qquad (2.5)$$

which can be used to analyze a multi-component signal $x = \sum_{i=1}^{N}\alpha_i x_i$, where $\alpha_i (i = 1 \ldots N)$ are constants. This property will be used in the algorithm to obtain periodic error amplitudes.

There are many types of wavelet functions available depending on the detection approach. Commonly used wavelets are the Haar, Daubechies, Meyer, Mexican Hat, and Morlet. Wavelets are selected based on the signal characteristic that can be extracted by the particular wavelet. For phase evaluation (i.e., for a sinusoidal model of periodic error, amplitude and phase are to be determined), the complex Morlet wavelet is suitable because it enables localization in both the time and frequency domains [45]. The frequency of the periodic error signal is located at the scale with the maximum wavelet coefficient and the phase information can be extracted based on the real and imaginary parts of this coefficient.

The complex Morlet wavelet is composed of a complex exponential multiplied by a Gaussian window (Fig. 3),

$$\psi^*\left(\frac{t-u}{s}\right) = \pi^{-\frac{1}{4}}e^{i2\pi f_0\frac{t-u}{s}}e^{-\frac{1}{2}\left(\frac{t-u}{s}\right)^2}, \qquad (2.6)$$