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Nonlinear modeling of compliant mechanisms incorporating circular flexure hinges with finite beam elements

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ABSTRACT

Modeling of compliant mechanisms incorporating flexure hinges is mainly focused on linear methods. However, geometrically nonlinear effects cannot be ignored generally. This work shows that nonlinear behavior plays an important role in the deformation and stress analysis, which consequently impacts the design of compliant mechanisms. In this study a nonlinear higher order finite beam element based modeling approach is presented strongly reducing the computation time of nonlinear models. Planar deformation and mechanical stress of a single circular flexure hinge under a wide range of loads is modeled and computed with the proposed approach. A comparison with a 3D-nonlinear finite element model shows very good agreement and validates the beam model. It is shown that the linear and nonlinear deformation behavior of a single flexure hinge deviate marginally so that linear modeling approaches are sufficient. Furthermore a planar displacement amplification mechanism incorporating circular flexure hinges is studied by means of the same method highlighting the distinct deviation of the behavior of the geometrically nonlinear model from its linear prediction. In conclusion the nonlinear behavior at the system level can not longer be neglected. Finally, a study shows that different designs of the displacement amplification mechanism are achieved when linear or nonlinear modeling approaches are applied.

1. Introduction

Flexure hinges allow rotation between two adjacent stiff members by the elastic deformation of a flexible connector. Because of their very precise, smooth motion compliant mechanisms based on flexure hinges find application for example in metrology instruments [1] and positioning systems for industry [2], where backlash, friction and wear are undesirable. The virtually limitless potential for miniaturization of compliant mechanisms established their broad use in microelectromechanical systems (MEMS).

The conceptual design of a compliant mechanism consist of the topological followed by the dimensional synthesis [3]. The topological synthesis is realized by means of topology optimization or kinematics based methods to determine the positions and interconnections of the flexure hinges within the design space. In both approaches a geometrically nonlinear analysis is commonly applied [4]. Goal of the challenging dimensional synthesis is the determination of size, geometry and shape of the flexure hinges and other parts of the mechanism. Many modeling approaches like the integration of the beam equations [5,6], inverse conformal mapping [7] and empirical methods [8,9] to describe the stiffness or compliance characteristics of flexure hinges have been proposed and validated. Also the finite element method with partly very large models has widely been employed [10]. These investigations are linear. The disagreement between a nonlinear topological and a linear dimensional synthesis is obvious but yet scarcely addressed in research. In a linear analysis of the system shown in Fig. 1, the force F causes a transverse deflection w but neglects the axial displacement u. This assumption holds true as long as the deflection is small. As the deflection becomes larger with an increasing force, the geometrically nonlinear effect can not longer be neglected [11]. The major drawback of flexure hinges is their limited capacity of rotation due to stress restrictions. Hence the optimization goal of developers of compliant mechanisms is to maximize the output motion [12]. Nevertheless linear models are applied. Also experimental data shows a deviation from the linear model for larger input forces, but a nonlinear effect is not suspected as cause [13].

The modeling of flexure hinges by finite beam elements has been proposed before but stayed limited to a linear analysis [14,15]. The Equivalent Beam Methodology (EBM) proposed by Zettl et al. is an approximation technique in which the flexure hinge is substituted by three straight finite beam elements to obtain a computationally efficient model with superior accuracy [14]. Nevertheless these desirable properties come with major drawbacks. The geometric

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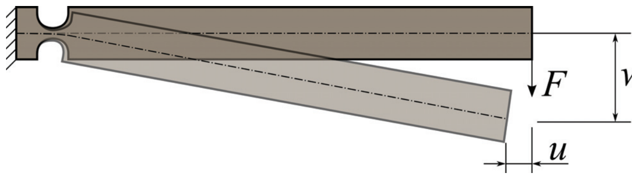

Fig. 1. Nonlinear deflection of a beam.

parameters of these three beam elements have to be obtained from a costly 3D analysis. Changing the loading condition or geometric parameters of the flexure hinge entails a recalculation of the 3D model. Taking advantage of advances in the development of finite element formulations for non-uniform beams [16], a 3-node finite beam element of variable cross-section for modeling circular flexure hinges has been proposed by Lobontiu [15]. Lobontiu's model takes shear deformation into account however, it is linear. Although nonlinear beam modeling approaches are readily available e.g. [17,18] they have rarely been employed to model flexure hinges. Boer et al. [19] present a substructuring method to obtain linear reduced mass and stiffness matrices of flexible members. From this basis a nonlinear superelement is generated for multibody analysis. However, this approach is not applied to the modeling and design flexible members with a continuous variation in cross-section like flexure hinges and the modal analysis, which is necessary to obtain the reduced mass and stiffness matrices, makes it impractical in applications where the geometry is updated continuously, e.g. in structural optimization.

In this work the linear modeling approach presented in [20], which simplifies the real deformation behavior of compliant mechanisms, is extended into the nonlinear field. The influence of geometrically nonlinear effects on the deformation and stress characteristics of planar compliant mechanisms is demonstrated. Since nonlinear analyses of 3D finite element models can consume an enormous amount of computational resources a sophisticated modeling approach based on an established geometrically nonlinear finite beam element procedure [18] is applied. This approach is validated by a simple benchmark with a single circular flexure hinge. Then a planar compliant displacement amplification mechanism incorporating circular flexure hinges is analyzed and the implications of nonlinear behavior on functionality and design of this compliant mechanism are discussed.

2. Nonlinear Euler–Bernoulli beam theory

Based on the Euler–Bernoulli hypothesis the governing equations of a planar geometrically nonlinear beam element are derived in accordance with [18] and a corresponding finite beam element procedure is developed. The implementation of a Timoshenko beam element is an alternative approach, especially for modeling short an compact beams. However, the main deformation of the flexure hinge occurs in the center of the flexure hinge as detailed 3D finite element analysis shows [21]. In the center of the flexure hinge the lengths to thickness ratio justifies an Euler–Bernoulli beam model.

2.1. Governing equations

In a Cartesian coordinate system (x, y, z) the displacement field (u_1, u_2, u_3) of a planar two dimensional geometrically nonlinear beam (cf. Fig. 2) is composed of axial displacements $u_0(x)$ and transverse deflections $w_0(x)$:

$$u_1 = u_0(x) - z_p \frac{dw_0(x)}{dx}, \qquad u_2 = 0, \qquad u_3 = w_0(x). \tag{1}$$

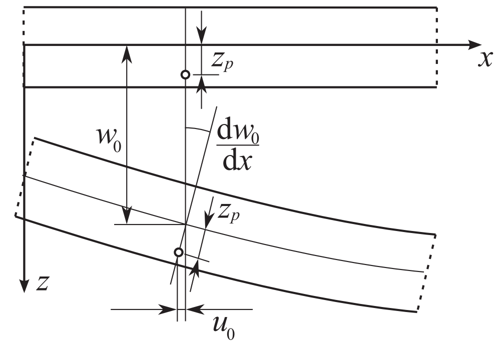

Fig. 2. Displacement field of an arbitrarily loaded and supported beam.

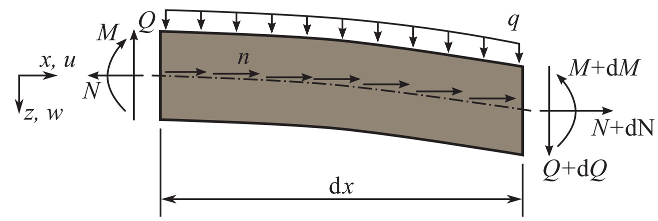

Fig. 3. A beam segment of length dx with internal and external forces.

Substituting the displacement field in Green's strain tenso

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) + \frac{1}{2}(u_{k,i}\, u_{k,j}) \tag{2}$$

and considering only the axial component an expression of the strain field

$$\epsilon_{11} = \underbrace{\frac{du_0}{dx} + \frac{1}{2}\left(\frac{dw_0}{dx}\right)^2}_{\epsilon_{11}^0} + z_p \underbrace{\left(-\frac{d^2 w_0}{dx^2}\right)}_{\epsilon_{11}^1} \tag{3}$$

$$= \epsilon_{xx}^0 + z_p\, \epsilon_{xx}^1 \tag{4}$$

where ϵ_{11}^0 is the axial strain and ϵ_{11}^1 is the curvature, is obtained. Other strain components are neglected. Applying Hooke's law $\sigma_{11} = E\epsilon_{11}$ with Young's modulus E, the internal forces

$$\begin{aligned} N &= EA\,\epsilon_{11}^0 \\ M &= EI\,\epsilon_{11}^1 \end{aligned} \tag{5}$$

are derived from the strains. Cross section area A and second moment of inertia I are functions of x as they contain the variable thickness of the flexure hinge. For clarity reasons the notation (x) is omitted in the following considerations. Performing the equilibrium of forces and moments for the infinitisimal beam segment of length dx in Fig. 3

$$\sum F_x = 0 = (N + dN) + n\, dx - N \tag{6}$$

$$\sum F_z = 0 = (Q + dQ) + q\, dx - Q \tag{7}$$

$$\sum M_y = 0 = (M + dM) - M - Q\, dx + N\, dx \frac{dw_0}{dx} + q\, dx\, (c_s\, dx) \tag{8}$$

where N, Q and M are the respective internal force variables and n and q are the respective external arbitrary distributed loads. Omitting higher order term and substituting Eq. (8) into (7) the governing equations of equilibrium

$$\begin{aligned} -\frac{dN}{dx} &= n \\ -\frac{d^2 M}{dx^2} - \frac{d}{dx}\left[N \frac{dw_0}{dx}\right] &= q \end{aligned} \tag{9}$$