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Dynamic modeling and model order reduction of compliant mechanisms

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ABSTRACT

In this work, a novel approach towards statical and dynamical modeling of compliant mechanisms is presented which serves as an origin for model order reduction procedures to provide small, efficient and accurate approximations. As 3D finite element modeling of compliant mechanisms results in very large-scale systems, both model reduction and real-time controlling of the mechanism are not possible. This and the fact that in compliant mechanisms mostly the flexure hinges contribute to the overall performance motivates a procedure which is based on partitioning the structure into elastically deformable hinges and rigid linkages. Unlike common modeling techniques, the reproduction of the non-negligible nonlinear behavior is assured and contributes to precise approximation as well as the fact that not only the flexure itself deforms but also adjacent structures. Furthermore, the proposed methodology is applicable to all kinds of flexure hinges with concentrated compliances and compliant mechanisms of complex geometric shape as well as spatial loading cases.

First, an analysis of the inserted flexure hinges yields the significantly deformed region of which corresponding master models with connecting nodes are created. With their geometric properties the mechanism is further divided into remaining stiff sections. Their spatial centroid location and moments of inertia about mass centroid are calculated and according point masses are generated. A finite element model of the mechanism is then developed by rigidly linking the master models with the point masses. Therewith, an accurate 3D model with appreciable less degrees of freedom arises, named significant region model. The system matrices **K**, **M** in combination with the input matrix **B** and the output matrix **C** yield a closed-form description of the mechanism.

In a second step, a considerable decrease of system size is performed by applying modern model order reduction techniques, namely Krylov subspace reduction. A comparison of the new two step approach with full 3D finite element modeling reveals only marginal deviations of less than 6% regarding the static displacements and less than 0.0001% relating to the frequency response of an exemplary mechanism with a substantial lower number of degrees of freedom.

1. Introduction

A novel methodology to enable feed units for small machine tools producing small workpieces is based on the application of compliant mechanisms (CM) being a monolithic structure composed of flexure hinges (FH) and their stiff connections, see citeWulfsbergsquare2010 and citeWulfsberg2013 for details. A prototype of such a feed unit is depicted in Fig. 1. Piezo-electric actuators (red arrows) provide the input displacements which are intensified by the CM providing a planar output trajectory (blue arrow) of the end effector (EE). Under the assumption of contact machining like milling, 3D loading cases are introduced to the CM and need to be considered.

Flexure hinges enable a relative motion between two stiff members by locally decreasing the bending stiffness by a variable cross-section yielding a compliant topology. Its elastic deformation and therewith its kinematic properties as well as its dynamic behavior depend on the geometric parameters and the material, respectively.

Because FHs are more accurate, simply scalable, cleaner, less noisy and cheaper in manufacturing and maintenance compared to classical pin joints [1], they are predestined especially for high precision, small scale applications like medical devices [2], micro grippers [3] and positioning stages [4]. On the other hand, certain disadvantages, namely limited rotation capability due to stress

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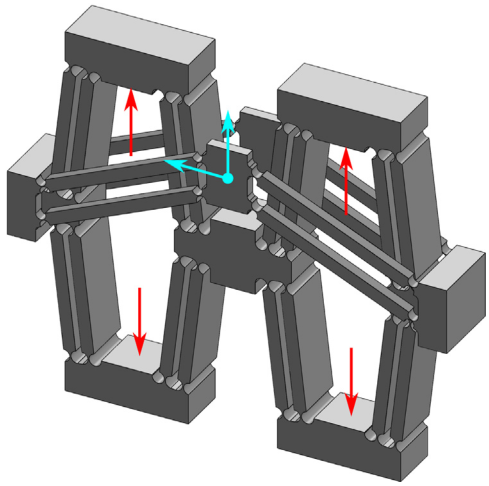

Fig. 1. Prototype of a compliant mechanism used as feed unit for ultra-precise applications with indicated piezo-electric actuators (red arrows) and planar output of the end effector (blue arrows). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 1
Summary of advantages and disadvantages of different modeling approaches.

Model	Advantages	Disadvantages
FEM	+ Accurate results	– Multitude of DOF
	+ Easy to generate	– Inapplicable for control
TBT	+ Analytical model	– Imprecise results
	+ Easy to implement	– Inapplicable for CMs
PRBM	+ Few DOF	– Imprecise results
	+ Easy to implement	– Inapplicable for CMs

concentration, topology generation of CMs and limitation due to fatigue strength are identified.

To embed the CM in a machine tool and drive this mechatronic device by piezo-electric actuators, a control system supplying appropriate input signals is necessary. A model of the CM in the form of a linear time invariant (LTI) system constitutes the basis of the control system requiring a low number (<100) of degrees of freedom (DOF) for real-time operations but being as accurate as possible which also includes geometric nonlinearities. A brief summary of drawbacks of common modeling approaches is given in Table 1 to motivate the novel approach for dynamic modeling of CM stated in this work. The abbreviation TBT denotes the Timoshenko beam theory and PRBM relates to the pseudo-rigid-body model.

Full 3D finite element models of complex CM may easily reach more than 1.000.000 DOF for mesh independent solutions and therewith are far too large whereas the approximation is highly accurate and nonlinearity may be included. Both finite beam and plane elements, as applied in [1], exhibit notable deviations compared to full 3D models as these do not reproduce the complex deformation state of flexure hinges.

The common PRBM approach, see [5–7] for details, does not have the ability to fulfill the requirements of ultra precise applications and is not suitable for CM of complex geometry with nonlinear behavior. The same holds for TBT. To authors knowledge, accurate and small-scale modeling techniques representing both the static and dynamic response of CMs incorporating nonlinear effects and spatial loading cases are not existing, especially for CMs of high complexity.

In order to overcome these problems of common approaches, a highly accurate model based on FEM is suggested which is reduced in size without considerable loss of accuracy.

This paper is composed of two parts. In the first part, a new modeling technique is proposed taking into account the significantly deformed region of loaded FHs to generate a geometrically specific and load independent master model which is described in Section 2. Assembling several master models via rigid connections to a CM results in an accurate model with considerably less DOF. In the second part therefore, a modern model order reduction approach is utilized where the size of the model is drastically diminished by a Krylov subspace reduction scheme for proportionally damped second order systems. This methodology is explained and validated by a numerical example in Section 3.

2. Significant region model: a novel modeling approach

In this section a new approach relying on FEM is presented, called significant region model (SRM), addressing the drawbacks of common modeling procedures commented in the previous section.

First, due to simplification, the method is presented by means of a single FH and is based on several assumptions which are listed below and illustrated in Fig. 2. Remark: Even though circular FHs are mostly utilized, polynomial FHs are investigated to avoid highly complex analytical expressions. The proposed approach is valid for all kinds of FH with concentrated compliance.

- Two symmetric, polynomial shaped cutouts form the FH.
- The minimal hinge height h_0, hinge length $2g$, polynomial hinge height $h(x) = h_0 + (h_* - h_0)\left(\frac{x}{g}\right)^p$ of order p, beam height $h = 2h_*$, beam depth b and beam length l define the geometric properties of the FH.
- The FH is located in the center of a straight beam.
- The beam is fixed at one end and free at the other.
- Due to simplification, only a shear force F_z at the free end is taken into account.
- Linear elastic deformations, isotropic material and small displacements are assumed.

The assumption of linear elastic deformations and small strains is acceptable investigating single FHs since they do not undergo

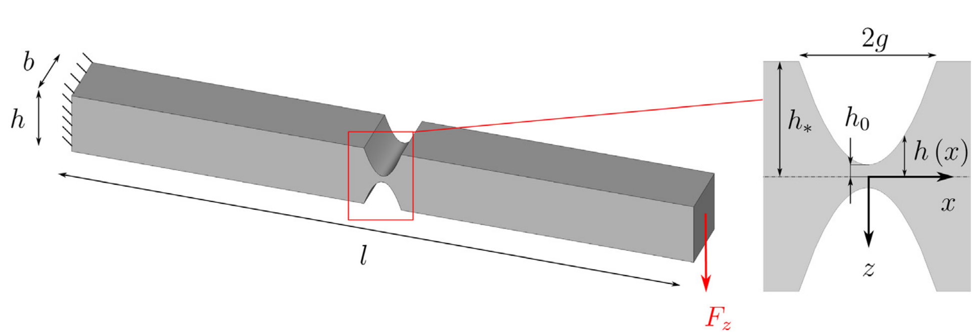

Fig. 2. Geometric parameters defining a FH.