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CIRP-1714; No. of Pages 4
ARTICLE IN PRESS
CIRP Annals - Manufacturing Technology xxx (2018) xxx–xxx
Contents lists available at ScienceDirect
CIRP Annals - Manufacturing Technology
journal homepage: http://ees.elsevier.com/cirp/default.asp

Ultimate capability of variable pitch milling cutters

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A R T I C L E I N F O

Keywords:
Tool geometry
Optimization
Chatter

A B S T R A C T

Variable pitch milling cutters intend to increase performance, but off-the-shelf cutters do not ensure this generally. Depending on the milling process they are selected for, they can perform better or even worse than uniform pitch cutters do. Improved performance can be guaranteed by considering the reflected dynamic behaviour of the machine/tool/workpiece system. This work presents the achievable upper and lower capability bounds by introducing so-called stabilizability diagrams of a hypothetical variable pitch milling cutter that is tuned continuously along the stability boundaries. Robustly tuned milling cutters are designed for selected spindle speed ranges, which are experimentally tested both under laboratory and industrial conditions.

1. Introduction

Variable pitch cutter is a good choice to improve productivity, but not in any extent as sometimes suggested by industrial commercial cutting tool brochures. In contrast, it is known [1] that the specific pitch angles need to be tuned to achieve real improvement. The tuning is based on taking into account the surface regeneration effect between subsequent cutting edges by considering corresponding delays in state variables [2]. The irregular distribution of flutes perturbs the destabilizing regenerative effect and has the potential to increase the productivity of milling operations.

The application of such cutters was already proposed by Hahn [3] in 1952, and many studies have been published on the problem since then. To maximize the stability in a certain range of spindle speeds, the cornerstone is the selection of optimal pitch angles between the cutting flutes [2]. The solution of [4] and its improvements in [5] provides graphical ways to tune the variable pitch angles. In [6], an initial stability calculation, while in [7], time domain simulations were used to obtain the best design parameters. Later, Budak [8] introduced a simple and effective analytical design method for finding optimal pitch angles, which was based on the zero order approximation (ZOA) of [9]. The effectiveness of the method was verified by experiments in [1]. In the work of Olgac and Sipahi [10], the cluster treatment of characteristic roots was introduced as a unique scheme to investigate the effect of variable pitch angles on dynamic stability of milling. An iterative method based on averaged dynamics and calculated by semidiscretization was presented in [11], which improved the results of [8]. While most of the above mentioned techniques are based on analytical/semianalytical approaches that do not take into account the time dependency of directional factors in milling operations, the time domain based solutions of [12,13] and [14] can overcome these limitations of ZOA.

The goal of this work is to go beyond the existing methods and to present the maximum what these special cutters can achieve. The idea is similar to the one introduced in [11], namely to build up a numerical iterative tuning scheme, but this time accurate time dependent dynamics and accurate frequency prediction are used. The relevance of precise variable pitch tuning is emphasized by presenting also the worst possible tuning that can occur as a result of accidental tool selection. The resultant ultimate tuning is validated by a laboratory and an industrial test. A special kind of period doubling (flip) dynamic stability loss is also observed, which is a direct consequence of the perturbed but still periodic regeneration effect of variable pitch cutters. This provides further qualitative validation of the applied model and methodology.

2. Milling model in short

In this section, a brief introduction is given to the mathematical formalism to be used in the optimization scheme of pitch angles. In standard mechanical models, the dynamics of the chip formation process is described by means of the empirical specific cutting force characteristics $\mathbf{f}(h)$, where the most relevant parameter is the chip thickness h [15]. Its momentary value $h \approx \mathbf{n}^{\mathrm{T}}(\Delta\mathbf{r} + v_f\tau\mathbf{e}_x)$ carries the dependency on time t through the local normal vector $\mathbf{n}$ at a given cutting edge and the regeneration $\Delta\mathbf{r}(t) = \mathbf{r}(t) - \mathbf{r}(t-\tau)$, with $\mathbf{r} = \mathrm{col}(x, y, z)$ denoting the mill centre position. The delay τ

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https://doi.org/10.1016/j.cirp.2018.03.005
0007-8506/© 2018 Published by Elsevier Ltd on behalf of CIRP.

Please cite this article in press as: Stepan G, et al. Ultimate capability of variable pitch milling cutters. CIRP Annals - Manufacturing Technology (2018), https://doi.org/10.1016/j.cirp.2018.03.005

connects the present and past tool motions to the secondary feed motion of velocity v_f in the direction $\mathbf{e}_x$.

In this manner, the cutting force acting on the tool of Z cutting edges is determined as the sum of the specific forces originated in the local momentary chip thicknesses h_i and accumulated along the axis z of rotation till the depth of cut a (see [16]):

$$\mathbf{F}(t,\mathbf{r}(t),\mathbf{r}_t(\tau_l)) = \sum_{i=1}^{Z}\int_0^a \boldsymbol{\gamma}_i(z,t,\mathbf{r}(t),\mathbf{r}_t(\tau_i))\mathrm{d}z, \tag{1}$$

$\gamma_i\,(z,t,\mathbf{r}(t),\mathbf{r}_t\,(\tau_i)) = -\,g\,(\varphi_{i,z,t})\,\mathbf{T}(\varphi_{i,z,t})\,\mathbf{f}(h_i(z,t,\mathbf{r}(t),\mathbf{r}_t\,(\tau_i)))/\sin\kappa_i\,(z)$ if $\varphi_{i,z,t}: = \varphi_i\,(z,t)$. The tool considered here has constant helix angle $\eta_i\,(z) = :\eta$, lead angle $\kappa_i\,(z) = :\kappa$, and radii $R_i\,(z) = :\,R = D/2$, so the corresponding trigonometric expressions are all included here in the geometric transformation matrix $\mathbf{T}$ (see [16]). The position angle of the ith edge is $\varphi_i\,(z,t) = \Omega\,t + \sum_{k=1}^{i-1}\varphi_{\mathrm{p},k} - (z/R)\tan\eta$, where $\varphi_{\mathrm{p},i}$'s are the pitch angles and Ω (rad/s) = $(2\pi/60)\,n$ (rpm) is the spindle speed. The actual form of the screen function g is given in [16] by means of the enter and exit angles of the milling process. Finally, the notation $\mathbf{r}_t\,(\tau_l) := \mathbf{r}(t-\tau_l)$ is used in (1) where τ_l corresponds to the lth tooth pass time $l = 1, \ldots, N_\tau$ where N_τ is the number of different constant delays ($N_\tau \le Z$; for example, $N_\tau = Z/2$ for alternated pitch angles and $N_\tau = Z$ for incremental ones).

The dynamics of the system is considered to be linear with damping ratios ξ_k, natural frequencies $\omega_{\mathrm{n},k}$ and mass normalized modal transformation matrix $\mathbf{U}$, which are determined by fitting the measured frequency response functions (FRFs) $\mathbf{H}(\omega)$ (m/N). Then, $\mathbf{H}(\omega) \approx \mathbf{U}(\mathrm{j}\omega\mathbf{I} - [\lambda_k])^{-1}\mathbf{U}^{\mathrm{T}}$ with $\lambda_k = -\,\omega_{\mathrm{n},k}\;\xi_k \pm \mathrm{j}\omega_{\mathrm{n},k}\,(1-\xi_k{}^2)^{1/2}$. The nonlinear periodic forcing in Eq. (1) induces time periodic stationary solution $\mathbf{r}_\mathrm{p}(t) = \mathbf{r}_\mathrm{p}(t+T_\mathrm{p})$ with principle time period $T_\mathrm{p} = T/N$, where $N = Z/\mathrm{rank}\,[\varphi_{\mathrm{p},(k+l-1)\bmod z}]_{k,l=1,\ldots,z}$ and $T = 2\pi/\Omega$. It is important to emphasize that T_p is not equal to the tooth passing period $T_Z = T/Z$. Solution $\mathbf{r}_\mathrm{p}$ can be calculated by boundary value solvers for DDEs [17].

The stability of stationary cutting is determined by introducing the perturbation $\mathbf{x}$ as $\mathbf{r} = \mathbf{r}_\mathrm{p} + \mathbf{x}$ in order to have the complex modal coordinates $\mathbf{u}$ by $\mathrm{col}(\mathbf{x},\dot{\mathbf{x}}) = \mathrm{col}(\mathbf{U}, \mathbf{U}[\lambda_k])\mathbf{u}$. This results in the linear time periodic DDE:

$$\dot{\mathbf{u}}(t) - [\lambda_k]\,\mathbf{u}(t) = \mathbf{U}^{\mathrm{T}}\sum_{i=1}^{Z}\mathbf{B}_i(t)(\mathbf{x}(t) - \mathbf{x}(t-\tau_i)), \tag{2}$$

where $\mathbf{B}_i\,(t) \equiv \mathbf{B}_i\,(t+T_\mathrm{p}) = -\int_a g\,(\varphi_{i,z,t})\,\mathbf{T}(\varphi_{i,z,t})\,\mathbf{K}_\mathrm{c}(h_{i,\mathrm{p}}(z,t))/\sin\kappa\;\mathrm{d}z$ the momentary cutting coefficient is $\mathbf{K}_\mathrm{c} = \mathrm{d}\mathbf{f}/\mathrm{d}h$ at the stationary chip thickness $h_{i,\mathrm{p}}(z,t) = \mathbf{n}^{\mathrm{T}}(\varphi_{i,z,t})(\mathbf{r}_\mathrm{p}(t) - \mathbf{r}_{\mathrm{p},t}\,(\tau_i) + v_f\,\tau_i\,\mathbf{e}_x)$. Determining the Floquet multipliers μ using a time domain based method like semidiscretization (SD) in [18], decaying/growing ($\sim\ln|\mu|$) and frequency ($\sim(\angle\mu + 2\pi k)$) properties of the solutions of (2) can be calculated, and the stability lobe diagrams (SLD) can also be determined in the parameter space. The same SLD can be obtained by the multi-frequency approach, too (see [19]).

Considering linear specific cutting force characteristics $\mathbf{f}(h) = \mathbf{K}_\mathrm{e} + \mathbf{K}_\mathrm{c}\,h$ and taking time average over the principle period T_p, a simplified single frequency representation can be derived (see details with ZOA in [8]):

$$\left(\mathbf{I} - a\,K_{\mathrm{c},t}\mathbf{H}\,(\omega)\sum_{l=1}^{N_\tau}(1 - \mathrm{e}^{-\mathrm{j}\,\omega\,\tau_l})\,\mathbf{A}_0/\sin\kappa\right)\mathbf{X}_0 = \mathbf{0} \tag{3}$$

with $\mathbf{A}_0 = \sin\kappa\int_{T_\mathrm{p}}\sum_i \mathbf{B}_i\,(t)\,\mathrm{d}t/a/T_\mathrm{p}/K_{\mathrm{c},t}$.

3. Tuning strategies

One-parameter optimizations are introduced (Fig. 1*e*) in order to compare the effectiveness of different algorithms. These showcases consider $Z = 4$ tooth cutters. First, alternated pitch angles [20] are used with $\varphi_{\mathrm{p},i} = (\varphi_{\mathrm{p},1},\ \varphi_{\mathrm{p},1} + \Delta\varphi_\mathrm{p},\ \varphi_{\mathrm{p},1},\ \varphi_{\mathrm{p},1} + \Delta\varphi_\mathrm{p})$, where $N = 2$ and $N_\tau = 2$. Second, the so-called linear variation is used with $\varphi_{\mathrm{p},i} = (\varphi_{\mathrm{p},1},\ \varphi_{\mathrm{p},1} + \Delta\varphi_\mathrm{p},\ \varphi_{\mathrm{p},1} + 2\Delta\varphi_\mathrm{p},\ \varphi_{\mathrm{p},1} + 3\Delta\varphi_\mathrm{p})$, where $N = 1$ and $N_\tau = 4$.

Classical tuning strategies are based on the regenerative phase shift ε_i defined between the just evolving and the past surface

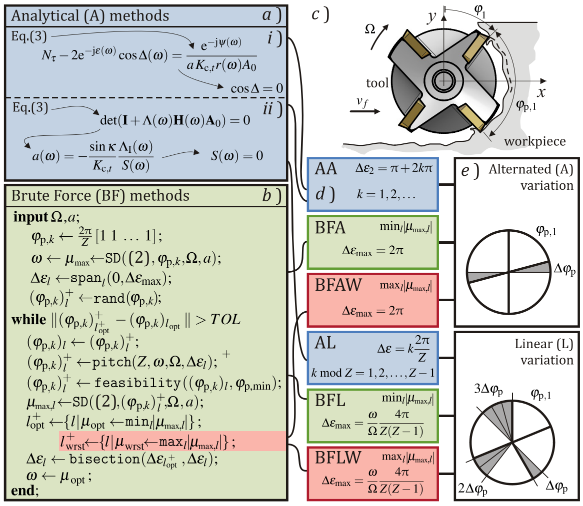

Fig. 1. The basic idea of classical analytical methods and the proposed numerical method are presented in *a*) and *b*), respectively. *c*) The sketch of the variable pitch cutter producing the ideal regenerative phase shift. Abbreviations for different methods for special pitch geometries *e*) are presented in *d*).

segments cut by the ith tooth. Thus, $\varepsilon_i = \omega\tau_i$, where $\tau_i = \varphi_{\mathrm{p},i}/\Omega$ (see Fig. 1*ab*), which gives $\varepsilon_i = (\omega/\Omega)\,\varphi_{\mathrm{p},i}$.

The original graphical method for edge-space-tuning presented by Slaviček in [4] is based on the one-dimensional (like x) version of (3) after substituting $H(\omega) = r(\omega)\mathrm{e}^{\mathrm{j}\psi(\omega)}$. On the unfolded (planar) geometric arrangement of the variable pitch milling tool, Slaviček derived a formula (see Fig. 1*ai*) by introducing an average regenerative phase $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$ and average phase difference $\Delta = (\varepsilon_2 - \varepsilon_1)/2$. He showed that a solution (graphical intersection) is least possible if $\cos\Delta \approx 0$. Based on this condition, the cutter can be designed analytically for the alternated geometry with methodology AA in Fig. 1*d*.

Tuning based on simplified but real circular tool geometry is presented by Budak in [8] with time-averaged dynamics based on the eigensolution of (3) for $\Lambda(\omega) = \Lambda_\mathrm{R}(\omega) + \mathrm{j}\Lambda_\mathrm{I}(\omega)$ (see Fig. 1*aii*). A chatter frequency ω-dependent solution is obtained analytically for the axial depth of cut a based on the assumption to be real valued. Mathematically, the solution is given by forcing $S(\omega) = \sum_l \sin\omega\tau_l$ to be minimal (or zero in extreme case). Budak's approach gives Slaviček's solution for alternated pitch variation; however, by neatly choosing the variation of the regenerative phase difference $\Delta\varepsilon$, S can be zero analytically for linear pitch variation, too, using the approach of Budak (see AL in Fig. 1*d*).

The presented brute force (BF) (see Fig. 1*b*) algorithm can be considered as a general method to increase stability boundary of cutting processes. The basic idea is to minimize the magnitude of the 'largest' Floquet multiplier $\mu_{\max}$, and to find the optimal μ_{opt} of μ for any given set of technological parameters. Here, the optimization is performed for the regenerative phase difference $\Delta\varepsilon$ and not for the actual geometric pitch angle differences $\Delta\varphi_\mathrm{p}$ as it was introduced in [11]. Keeping feasibility conditions, the optimum is found by bisection algorithm up to a given tolerance.

Both alternated and linear variation topology can be optimized with BF algorithms denoted by BFA and BFL, respectively, among the methods in Fig. 1*d*. In order to highlight the importance of precise pitch angle tuning further, worst case scenarios are also presented for both topologies with BFAW and BFLW in Fig. 1*d*, where the largest multiplier in magnitude is maximized (μ_{wrst}).

4. Stabilizability diagrams

In order to have comparable results, all methods introduced in Fig. 1*d* were computed using SD. After having the multipliers, the vibration frequencies ω are calculated, then the method has been applied analytically in AA and AL, and numerically in all BF cases. This way, the best achievable performance within stability can be