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Studies about the use of semicircular beams as hinges in large deflection planar compliant mechanisms

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ABSTRACT

Conventional hinge designs in planar compliant mechanisms have a limited deformation range because of the high stresses induced during deflection. To improve the range of motion of these mechanisms, hinges that allow for large displacement are highly desirable. This paper explores the use of curved beams as large displacement hinges in planar compliant mechanisms. To facilitate design, analytic expressions that predict deflections under different types of loads are introduced. These expressions are used in pseudo rigid link models of compliant mechanism designs. Predictions made by the analytic expressions are compared with the results of FEA simulations. To validate the proposed models, two planar compliant mechanism designs were prepared and experimental measurements of deflections under loads were made. Overall, results showed that analytic models and FEA predictions lie within 10% of experimental data for the planar mechanism geometry in which pseudo rigid motion models apply. FEA models of the second case, a more complex mechanism, make predictions that lie within 15% of experimental measurements. Results and ways to improve accuracy of models and designs are discussed at the end of the article.

1. Background

In planar compliant mechanisms, weak sections are designed and built into a structure to produce specific motions of rigid parts of the mechanism. The weaker sections are commonly referred to as “elastic hinges”, and replace the revolute joints that are used in conventional rigid link mechanisms. Mechanism compliance is a direct result of the elastic response of the joints.

Compliant mechanisms are inherently insensitive to the nonlinearities and inaccuracies caused by conventional joints, a characteristic that makes them very attractive for high precision applications. To maintain geometric integrity, however, stresses induced by the deformation of the compliant mechanism have to be kept below the yield strength of the material. This limits the range of motion of compliant mechanisms, which in turn constitutes a limitation for their application.

A number of studies have proposed hinge designs that improve the range of motion of compliant mechanisms. Of particular interest is the work of Trease [1], who introduced a variety of designs that allow for large deformations, and described some of the challenges these designs need to overcome. In particular, Trease explains that besides an improved range of motion, the ideal design should reduce axis drift and stress concentration, and increase off-axis stiffness.

This article introduces a hinge design that relies on semi circular beams, which is particularly suitable for planar mechanisms. The proposed design combines a relatively well-localized center of rotation, comparable to those of conventional hinges, with deflections that are more characteristic of long beams. To facilitate design, analytic models that can be used for mechanism synthesis in the manner proposed by Howell [2] are introduced and validated with FEA models. In general, good agreement was found. Experimental tests with prototype planar compliant mechanisms that use semicircular beams as hinges are presented at the end of the article.

2. Literature review

There is an extensive literature associated with common applications of compliant mechanisms. The works by Howell [3], Smith [4] and Lobontiu [5] constitute excellent references for the design and analysis of compliant mechanisms.

Traditionally, hinge designs have been used to allow small deflection of mechanisms and analysis has typically focused on the deformation of the hinge when subject to different types of

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Nomenclature

Coordinates and displacements

δ	deflection at tip caused by force
u, v	tip displacements, free end
φ	span angle, semicircular beam
$\alpha, \beta, \gamma, \psi, \lambda$	internal angles in four bar mechanism
θ^r	generalized input angle four bar mechanism
$\theta_{in}, \theta_{out}$	input, output angles in mechanism
θ_2, θ_3	internal angles, planar mechanisms toggle mechanism
$\Delta\theta_{pin}$,	rotation in joints
θ	angle of curvature at tip of semicircular beam

Forces and moments

P	radial force at tip of beam
Q	normal force at tip of beam
Mo	external Moment
V	shear force, internal
N	normal force, internal
M	normal moment, internal
F_{in}	input force, mechanism

Material properties and geometric parameters

E	Young' Modulus
I	first moment of area
μ	Poisson' ratio
Ax	cross sectional area of beam
k_s	form correction factor for shear
K_t, K_2, K_3	torsional spring constant
k	radius of curvature
Ri	initial radius, semicircular beam
r_o, r_1, r_2, r_3, r_4	link lengths
a, b, c, d, f, g	mechanism dimensions
x_{in}, x_{out}	input and output displacements, toggle mechanism

Miscellaneous

U	energy of deformation
W	work

loads. In a classic article, Paros and Weisbord [6] introduced circular hinge design formulas and guidelines. Over the years, their work has been reviewed and compared with the results of FEA models by Yong [7], and augmented by the analysis of the effects caused by typical manufacturing errors of the hinge geometry by Ryu [8]. Further hinge designs have been introduced and analyzed by Smith [9], who presented semi elliptic designs and by Tian [10], who analyzed V-shaped hinges. Lobontiu et al. [11] presented equations for the prediction of deformation and displacement of conic section hinges under different loads. In another article Lobontiu [12] introduced formulas for the analysis of symmetric two-axis flexure hinges.

The need for large deflection hinges has also been discussed extensively in the literature, and efforts have been made to develop and model their performance. In addition to the work of Trease [1] discussed before, Howell [2] introduced the concept of Pseudo-Rigid Body to model the deflection path of end loaded cantilever beams subject to large deflections. In this approach, flexible links are replaced by a combination of rigid links pivoted at joints that are augmented by torsional springs. An important issue during the development of the models is to locate the center of rotation of the pseudo rigid links. Yu et al. [13] extended this approach to model dynamic equations of planar compliant mechanisms. The application of large deflection beams for the design of bistable mechanisms is explored by Jensen [14] and Masters [15]. As their name implies, bistable mechanisms present two stable positions, at local minima of stored energy, and are commonly found in valves and switches [16]. Jensen [14] presented analytical techniques to simplify modeling of specific configurations, while Masters [15] developed three degree of freedom pseudo rigid models of self-retracting bistable mechanisms. Sönmez [17] proposed the use of buckling beams and arcs to design compliant mechanisms. The idea is to use the buckling characteristic of long beams and arcs to produce dwell in five bar compliant mechanisms. As shown in these studies, compliant mechanisms are particularly suitable for micro actuators because they avoid problems associated with the use of pinned joints and because they can return to their initial states without the need for external forces.

In regards to specific applications, there is an extensive list of reports of designs that combine actuators and compliant mechanisms to either amplify or scale down output motion. Three cases that represent different families of mechanisms and which expose and address typical design and performance issues are discussed here. In the first case, Juuti et al. [18] combined piezoelectric actuators with flexible hinge-long link mechanisms to produce displacements of about 1 mm. Actuator structures measured about 50 mm. Mathematical modeling of their mechanisms assumed flexible pivots and rigid links, and predicted amplification ratios of about 10:1. Experiments with prototypes reported voltage–displacement hysteresis, and amplifications of about 16:1. Authors attributed the difference with respect to analytic predictions to deviations in the geometry of the prototype as well as stresses generated during the manufacture and assembly of the mechanism. Ha et al. [19] designed a planar micropositioning stage that combined a lead zirconate titanate (PZT) actuator with flexure hinge (Scott–Russell or linear) mechanisms. They used Taguchi techniques to optimize their design based on FEA modeling of the structure. Their goal was to obtain the maximum amplification factor as a function of the geometric characteristics of the mechanism. Their simulations reported a maximum amplification ratio of 2:1, while their experimental data showed a 2.1:1 ratio. Compared to the design by Juuti, Ha' mechanism has a smaller amplification factor because of the use of hinges as opposed to the beams used by Juuti. Ha' modeling of the mechanism was based on FEA and their predictions matched their experimental work with good accuracy. Lu et al. [20] present a planar mechanism that also used PZT actuators and flexible hinge-rigid links in a three-degree of freedom planar mechanism. That is, linear motion in the X–Y plane plus rotation around the Z plane can be controlled with such a stage. They used Howell' pseudo rigid model to predict the behavior of the mechanisms and compared these results to FEA simulations of their mechanism. Predictions made by the linear models were very similar to those made with non-linear formulation, but both differed significantly (by as much as 100%) from those predicted by the FEA models. FEA models in turn matched experimental data much more closely. They attributed the error to the limitations of the analytic models, such as inability to account for the compliance of the joints in out of plane directions. The difficulty in isolating the exact point of rotation of each joint in the pseudo rigid models could have also affected the accuracy of their results. Finally, Culpepper [21] proposed a slightly different approach than the previous studies, which combined low cost, large travel range, linear actuators with compliant mechanisms that reduced the displacement of the actuators to produce high precision motion in three dimensions.

From the previous discussion, compliant mechanisms are highly desirable for precision applications. Conventional hinges, which have the advantage of ease of manufacture and robustness, typically display shapes that are subject to stress concentration: leaf, elliptic, circular and more recently, V-shapes. This fact limits their range of deformation. Beams have been used as large deflection links to expand the range of application of compliant mechanisms.