G Model
CIRPJ-298; No. of Pages 11
ARTICLE IN PRESS

CIRP Journal of Manufacturing Science and Technology xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

CIRP Journal of Manufacturing Science and Technology

journal homepage: www.elsevier.com/locate/cirpj

Stability analysis in milling of flexible parts based on operational modal analysis

Bartosz Powałka [a,*], Krzysztof Jemielniak [b,1]

[a] Institute of Manufacturing Engineering, Faculty of Mechanical Engineering and Mechatronics, West Pomeranian University of Technology, Szczecin, Piastów 19, 70-310 Szczecin, Poland
[b] Faculty of Production Engineering, Warsaw University of Technology, Narbutta 86, 02-524 Warszawa, Poland

ARTICLE INFO

Article history:
Available online xxx

Keywords:
Stability
Milling
Operational modal analysis

ABSTRACT

The classic approach to stability analysis in milling is based on the frequency response function (FRF) of the system obtained by the impact test. Application of this technique for a flexible workpiece with variable dynamics such as thin-walled structures may be difficult or impossible because the modal parameters of a part change due to material removal or tool position. Besides the workpiece vibration are significant compared to that of the tool. Therefore precise determination of the varying FRF is vital. It can be achieved by application of operational modal analysis based on workpiece acceleration measured during machining. The novelty of the proposed procedure lies in using the cutting force model for scaling modal residues that enables constructing stability lobes diagram from output-only data.

1. Introduction

Milling of flexible parts is a challenging task due to the possible occurrence of regenerative chatter. Generally, determination of the stability limit requires the cutting force model and transfer functions of the machine tool-workpiece system. The former is usually described by the mechanistic force model [1] and the latter is represented by the frequency response function (FRF). Typically, the FRF is identified through the impact test. This might be difficult or even impossible for case of flexible workpiece, when the FRF varies along the tool path and in time due to reduction of the workpiece mass. To overcome this difficulty, measuring the workpiece dynamics at different positions (e.g. [2,3]), or FEM analysis (e.g. [4,5]) are applied. Attractive alternative might be an operational modal analysis (OMA) which allows to extract modal parameters from the acceleration signal measured during actual machining. Zaghbani and Songmene [6] proposed an algorithm of the OMA limited to the estimation of modal damping and natural frequency, whereas modal residues were not identified. This made synthesis of the FRF using only output data impossible. Identification of the transfer function from the milling experiments was also proposed by Suzuki et al. [7]. Their algorithm requires performing multiple tests at unstable cutting conditions. The algorithm allows for estimation of modal mass and natural frequency whereas damping must be assumed a priori to construct the transfer function. Also Kruth et al. [8] developed an inverse methodology to identify dynamic system parameters directly by making a few experiments during cutting. In [9] a modified stability criterion to utilize unscaled modal residues identified using acceleration signals measured during cutting was proposed. Such an approach provided information on the stable spindle speeds whereas limit depth of cut was not accurately determined.

In this paper, modal parameters are estimated using acceleration signals measured during stable cutting. A new approach is proposed to evaluate quantitatively a flexible workpiece dynamics using only output data. Modal residues are scaled in the frequency domain using amplitudes of the acceleration signal and the harmonic component of the modeled cutting force. Such the procedure allows not only to find location of stability pockets but also to evaluate limit depth of cut.

2. Method of generation of stability lobes diagram based on operational data

The proposed method consists of the following steps:

1. Measurement of acceleration signals in the close vicinity of the tool–workpiece interface.

* Corresponding author. Tel.: +48 91 449 4930; fax: +48 91 449 4442.
E-mail addresses: bartosz.powalka@zut.edu.pl (B. Powałka), k.jemielniak@gmail.com (K. Jemielniak).
[1] Tel.: +48 22 234 8656; fax: +48 22 8490 285.

http://dx.doi.org/10.1016/j.cirpj.2014.11.003
1755-5817/

2. Extraction of modal parameters from acceleration signal, i.e., modal damping, natural frequency using OMA.
3. Computing scaled modal residues on the basis of measured acceleration, analytical representation of the cutting forces and estimated modal damping and natural frequency – synthesis of the frequency response function.
4. Generation of stability lobes diagram using a synthesized frequency response function.

2.1. Estimation of natural frequency

In this paper, a modified Least Squares Complex Exponential (LSCE) method in the presence of harmonic excitation was applied in order to extract modal parameters from the output-only data [10]. The operational version of the LSCE method is based on the Natural Excitation Technique [11]: if the system is excited by stationary white noise, the auto- and cross-correlation functions between the response signals are similar to the impulse response signal and can be expressed in a discrete time domain as:

$$R(k\Delta t) = \sum_{r=1}^{N} C_r \exp(s_r k\Delta t) + C_r^* \exp(s_r^* k\Delta t) \qquad s_r = -\omega_r \xi_r + i\omega_r\sqrt{1-\xi_r^2} \tag{1}$$

or in frequency domain

$$S_{xx} = \sum_{r=1}^{N} \frac{C_r}{i\omega - s_r} + \frac{C_r^*}{i\omega - s_r^*} \tag{2}$$

where N is the number of modes in the system, C_r is a modal constant associated with the rth mode, ω_r and ξ_r are the rth non-damped natural frequency and damping ratio, S_{xx} is the power spectral density of the response signal. In typical LSCE OMA procedure one has to find coefficients β from the overdetermined system of equations:

$$\begin{bmatrix} R_0 & R_1 & \cdots & R_{2N-1} \\ R_1 & R_2 & \cdots & R_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ R_p & R_{p+1} & \cdots & R_{p+2N-1} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{2N-1} \end{bmatrix} = -\begin{bmatrix} R_{2N} \\ R_{2n+1} \\ \vdots \\ R_{2N+p} \end{bmatrix} \tag{3}$$

Once these coefficients are computed the complex eigenvalues s_r are found by computing roots of the polynomial

$$\beta_0 + \beta_1 V_r^1 + \beta_2 V_r^2 + \cdots + \beta_{2N-1} V_r^{2N-1} + V_r^{2N} = 0 \tag{4}$$

where $V_r = \exp(s_r \Delta t)$.

In the presence of harmonic excitation, in addition to the random loads, the correlation functions will include non-decaying components. These components will be identified as poles with a very small damping and the user will have to eliminate them by assuming threshold for damping or it must be performed manually as shown for instance by Zaghbani and Songmene [6]. To cope with this difficulty, Mohanty and Rixen [10] proposed a modified version of the LSCE which forces the modal solution to have non-damped poles with natural frequencies equal to excitation frequencies. Assuming m harmonic frequencies $\omega_1, \omega_2, \ldots \omega_m$ a system of Eq. (3) is expanded to:

$$\begin{bmatrix} R_0 & \ldots & R_{2m-1} & R_{2m} & \ldots & R_{2N+2m-1} \\ \vdots & \ldots & \vdots & \vdots & \ldots & \vdots \\ R_p & \ldots & R_{p+2m-1} & R_{p+2m} & \ldots & R_{p+2N+2m-1} \\ 0 & \ldots & \sin(\omega_1(2m-1)\Delta t) & \sin(\omega_1 2m\Delta t) & \ldots & \sin(\omega_1(2N+2m-1)\Delta t) \\ 1 & \ldots & \cos(\omega_1(2m-1)\Delta t) & \cos(\omega_1 2m\Delta t) & \ldots & \cos(\omega_1(2N+2m-1)\Delta t) \\ \vdots & \ldots & \vdots & \vdots & \ldots & \vdots \\ 0 & \ldots & \sin(\omega_m(2m-1)\Delta t) & \sin(\omega_m 2m\Delta t) & \ldots & \sin(\omega_m(2N+2m-1)\Delta t) \\ 1 & \ldots & \cos(\omega_m(2m-1)\Delta t) & \cos(\omega_m 2m\Delta t) & \ldots & \cos(\omega_m(2N+2m-1)\Delta t) \end{bmatrix} \begin{bmatrix} \beta_0 \\ \vdots \\ \beta_{2m-1} \\ \beta_{2m} \\ \vdots \\ \beta_{2N+2m-1} \end{bmatrix} = -\begin{bmatrix} R_{2N+2m} \\ \vdots \\ R_{P+2N+2m} \\ \sin(\omega_1(2N+2m)\Delta t) \\ \cos(\omega_1(2N+2m)\Delta t) \\ \vdots \\ \sin(\omega_m(2N+2m)\Delta t) \\ \cos(\omega_m(2N+2m)\Delta t) \end{bmatrix} \tag{5}$$

The frequency of harmonic excitation is assumed to be known a priori as it is in the case of milling. In the present study, harmonic components were assumed to be multiples of the spindle rotation frequency. Application of this method simplifies elimination of virtual modes from the stabilization diagram because poles corresponding to the harmonic excitation have zero damping. Stabilization diagram shows the evolution of frequency, damping, as the number of modes is increased. The poles (s_r) that do not change significantly are selected for further analysis.

2.2. Computing scaled modal residues

The relationship between power spectral density S_{xx}, frequency response function G and power spectral density of the excitation forces can be expressed as:

$$S_{xx} = G(i\omega) S_{FF} G(i\omega)^H \tag{6}$$

where the frequency response function is

$$G(i\omega) = \sum_{r=1}^{N} \frac{A_r}{i\omega - s_r} + \frac{A_r^*}{i\omega - s_r^*} \tag{7}$$

When the system is excited by stationary white noise with variance σ^2, the power spectral density function S_{xx}, takes form:

$$S_{xx} = 2\sigma^2 \Delta t G(i\omega) G(i\omega)^H \tag{8}$$

Eqs. (5), (4) and (2) relate modal residue A_r to modal constant C_r as

$$C_r = 2\sigma^2 \Delta t \frac{A_r A_r^H}{2\omega_r \xi_r} \tag{9}$$

which is valid when the damping is light.

Since the variance of the excitation forces σ^2 is unknown, modal residues are not known exactly and, therefore, an FRF matrix cannot be accurately synthesized. The scaling factor may be found through the analytical model [12] of the tested structure or from the mass change effect [13]. These approaches require either an accurate model of the structure or carrying out an additional test after introducing mass changes at the chosen measurement point. Both requirements are not feasible in an industrial environment.

Scaled modal *residues* A_r can be approximated utilizing the fact that the response of the system in the frequency domain is a linear