Nuclear Inst. and Methods in Physics Research B 421 (2018) 18–26

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

X-ray radiation generated by a beam of relativistic electrons in composite structure

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ARTICLE INFO

Keywords:
Relativistic electron
Coherent radiation
Composite structure

ABSTRACT

The dynamic theory of coherent X-ray radiation generated by a beam of relativistic electrons in the three-layer structure consisting of an amorphous layer, a vacuum (air) layer and a single crystal has been developed. The phenomenon description is based on two main radiation mechanisms, namely, parametric X-ray radiation (PXR) and diffracted transition radiation (DTR). The possibility to increase the spectral-angular density of DTR under the condition of constructive interference of the transition radiation waves from different boundaries of such a structure has been demonstrated. It is shown that little changes in the layers thicknesses should not cause a considerable change in the interference picture, for example, the transition of constructive interference into destructive one. It means that in the considered process the conditions of constructive interference are enough stable to use them for increasing the intensity of X-ray source that can be created based on the interaction of relativistic electrons with such a structure.

1. Introduction

When an electron moving rectilinearly with a steady speed crosses a boundary between two media the transition radiation (TR) arises along the electron velocity [1]. A great interest in transition radiation of relativistic electron is due to the possibility of its application as an alternative source of X-ray radiation [2]. When a charged particle crosses a single-crystal plate, TR photons emitted at the plate entrance surface might diffract on a system of parallel atomic planes of the crystal. Such radiation characterized by narrow spectral range propagating in the direction of Bragg scattering is known as diffracted transition radiation (DTR) [3,4]. TR from the entrance surface contributes to DTR, while TR from the outlet surface of the plate does not take part in the DTR formation. Hence, in the resulting DTR there is no interference of these two types of TR.

In addition to TR, when relativistic electron crosses single crystal, its interaction in a crystal bulk with parallel atomic planes results in generating parametric X-ray radiation (PXR) [5–7]. PXR photons are emitted in the direction of the Bragg scattering and propagate together with DTR photons. The dynamic theory of coherent X-ray radiation by relativistic electrons in a crystal is developed for general case of electron Coulomb field reflection asymmetric with respect to the target surface [8–10]. In that case the system of parallel reflecting layers in the target has an arbitrary non-zero angle with the target surface.

Traditionally, radiation of relativistic electrons is analyzed separately for amorphous, crystalline or multilayer targets. Till now coherent radiation of relativistic electrons in composite targets has not been theoretically evaluated, while experimental studies on generation of coherent X-ray radiation [11–15] in composite structures have revealed an essential growth of the intensity of DTR yield with the increase of the number of boundaries (layers). The influence of the electron beam divergence on the PXR and DTR spectral-angular density has been examined in [16,17].

In the present work, we present our analysis on the process of coherent radiation by a beam of relativistic electrons in a complex target formed by the two amorphous and one single-crystal layers in two-wave approximation for dynamic diffraction theory.

2. Geometry of radiation process

Let us consider a beam of relativistic electrons passing through a three-layer structure consisted of two layers of amorphous media and one single-crystal layer (Fig. 1a) with different thicknesses c, a and b respectively. We will use the Planck's system of units (h/2π = c = 1, where h is the Planck's constant, c is the light velocity).

Below we denote the dielectric susceptibility of amorphous media as χ_c and χ_a, the average dielectric susceptibility of the crystal as χ_0 and the coefficient of Fourier expansion of the crystal dielectric susceptibility over the reciprocal lattice vectors **g** as

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https://doi.org/10.1016/j.nimb.2018.02.013
Received 5 November 2017; Received in revised form 12 February 2018; Accepted 13 February 2018

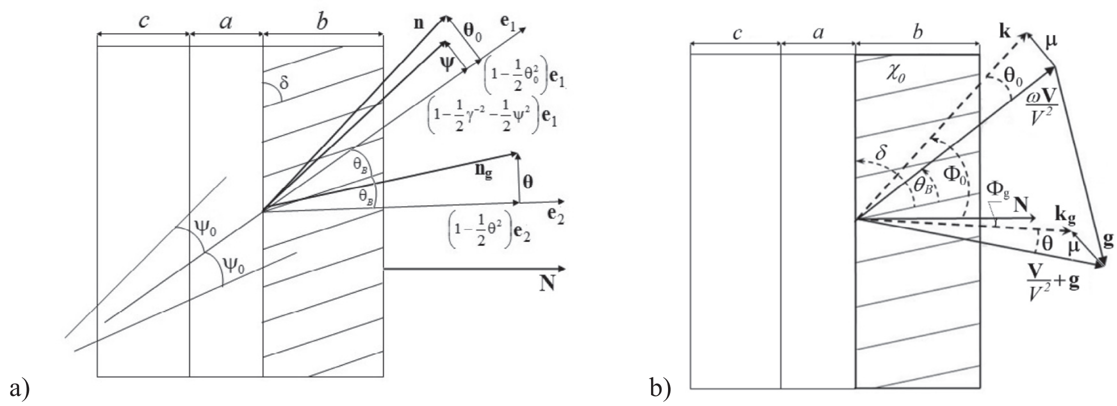

Fig. 1. Radiation process geometry (a). Geometry of the wave diffraction process in single-crystal layer (b).

$$\chi(\omega,\mathbf{r}) = \sum_{\mathbf{g}} \chi_{\mathbf{g}}(\omega)\exp(i\mathbf{gr}) = \sum_{\mathbf{g}} (\chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega))\exp(i\mathbf{gr}), \quad (1)$$

while the angular variables ψ, θ и θ_0 as consistent with the definition for the velocity vector $\mathbf{V}$ of relativistic electron and unit vectors $\mathbf{n}$ (in the direction of the photon momentum emitted along the electron velocity) and $\mathbf{n_g}$ (in the Bragg scattering direction):

$$\mathbf{V} = \left(1-\tfrac{1}{2}\gamma^{-2}-\tfrac{1}{2}\psi^2\right)\mathbf{e}_1 + \psi, \quad \mathbf{e}_1\psi = 0$$
$$\mathbf{n} = \left(1-\tfrac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \theta_0, \quad \mathbf{e}_1\theta_0 = 0, \quad \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B,$$
$$\mathbf{n_g} = \left(1-\tfrac{1}{2}\theta^2\right)\mathbf{e}_2 + \theta, \quad \mathbf{e}_2\theta = 0 \quad (2)$$

Here θ is the radiation angle counted from the detector axis $\mathbf{e}_2$, ψ is the angle of electron deviation from the original beam axis $\mathbf{e}_1$, θ_0 is the angle between the propagation direction of incident pseudo photon in electron Coulomb field and the beam axis $\mathbf{e}_1$, $\gamma = 1/\sqrt{1-V^2}$ is the Lorentz factor. The angular variables are considered as a sum of components parallel and perpendicular to the plane of the figure:

$$\theta = \theta_{||} + \theta_{\perp}, \quad \theta_0 = \theta_{0||} + \theta_0\perp, \quad \psi = \psi_{||} + \psi_{\perp}$$

At first, we will consider the radiation by single electron in the beam crossing three-layer structure at the angle $\psi(\psi_{\perp},\psi_{//})$ to electron beam axis $\mathbf{e}_1$.

3. Radiation amplitude

The X-ray wave propagation in the single-crystal medium will be considered within the scope of two-wave approximation of dynamical diffraction theory by analogy with the work [9].

As the electromagnetic field excited by relativistic electron is practically transverse in X-ray frequency band the incident $\mathbf{E}(\mathbf{k},\omega)$ and diffracted $\mathbf{E}(\mathbf{k}+\mathbf{g},\omega)$ in the crystal electromagnetic waves are characterized by two amplitudes with different values of polarization

$$\mathbf{E}(\mathbf{k},\omega) = E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)},$$
$$\mathbf{E}(\mathbf{k}+\mathbf{g},\omega) = E_{\mathbf{g}}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)} + E_{\mathbf{g}}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)}$$

where the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to the vector $\mathbf{k} = k\mathbf{n}$ and vectors $\mathbf{e}_1^{(1)}$ and $\mathbf{e}_1^{(2)}$ are perpendicular to the vector $\mathbf{k_g} = \mathbf{k}+\mathbf{g} = k_{\mathbf{g}}\mathbf{n_g}$. The vectors $\mathbf{e}_0^{(2)}$ and $\mathbf{e}_1^{(2)}$ lie in the plane of $\mathbf{k}$ and $\mathbf{k_g}$ vectors ((π-polarization), but $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ vectors are perpendicular to that plane (σ-polarization).

Within the scope of two-wave approximation of diffraction dynamic theory the system of equations for the amplitude of the wave field strength can be expressed in the following form

$$\begin{cases}(\omega^2(1+\chi_0)-k^2)E_0^{(s)} + \omega^2\chi_{-\mathbf{g}}C^{(s)}E_{\mathbf{g}}^{(s)} = 8\pi^2 ie\omega\mathbf{e}_0^{(s)}\mathbf{V}\delta(\omega-\mathbf{kV}),\\ \omega^2\chi_{\mathbf{g}}C^{(s)}E_0^{(s)} + (\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)} = 0,\end{cases} \quad (3)$$

where

$$C^{(s)} = \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = \cos 2\theta_B$$
$$\mathbf{e}_0^{(1)}\mathbf{V} = \theta_{\perp}-\psi_{\perp} \equiv \Omega^{(1)}, \quad \mathbf{e}_0^{(2)}\mathbf{V} = \theta_{//} + \psi_{//} \equiv \Omega^{(2)},$$
$$\chi_{\mathbf{g}} = \chi'_{\mathbf{g}} + \chi''_{\mathbf{g}}, \quad \chi'_{\mathbf{g}} = \chi'_0(F(g)/Z)(S(\mathbf{g})/N_0)\exp(-g^2u_\tau^2/2), \quad \chi''_{\mathbf{g}} = \chi''_0\exp\left(-\tfrac{1}{2}g^2u_\tau^2\right)$$

where $\chi_0 = \chi'_0 + i\chi''_0$ is average dielectric susceptibility, F(g) is form-factor of an atom contained Z electrons, $S(\mathbf{g})$ is structural factor of unit cell in single-crystal layer contained N_0 atoms, u_τ is the mean-square amplitude of thermal atomic oscillations in the crystal. In the present work, the X-radiation frequency region is considered: $(\chi'_{\mathbf{g}} < 0)$, $(\chi'_0 < 0)$. The system of Eq. (3) under $s = 1$ describes the fields of σ- polarization, and under s = 2 the fields of π-polarization.

It should be noted, that the system (3) describes the fields of X-ray waves and Coulomb fields of relativistic electrons both in vacuum (under $\chi_{\mathbf{g}} = 0$, $\chi_0 = 0$) and in amorphous media (under $\chi_{\mathbf{g}} = 0$ and $\chi_0 = \chi_a$ or $\chi_0 = \chi_c$).

In Fig. 1b a scheme of the wave diffraction in single crystal is shown, where $\mu = \mathbf{k}-\omega\mathbf{V}/V^2$ is the component of virtual photon momentum perpendicular to particle velocity $\mathbf{V}$ ($\mu = \omega\theta_0/V$, where $\theta_0 \ll 1$ is the angle between vectors $\mathbf{k}$ and $\mathbf{V}$), θ_B is Bragg angle the modulus of vector $\mathbf{g}$ can be expressed by the Bragg angle θ_B and Bragg frequency ω_B: $g = 2\omega_B\sin\theta_B/V$. The angle between the vector $\omega\mathbf{V}/V^2$ and the wave vector $\mathbf{k}$ of incident wave is denoted as θ_0 and the angle between the vector $\omega\mathbf{V}/V^2 + \mathbf{g}$ and wave vector $\mathbf{k_g}$ of diffracted wave as θ.

The magnitude of the wave vector of free photons in amorphous media $k_a = \omega\sqrt{1+\chi_a}$ and $k_c = \omega\sqrt{1+\chi_c}$ can be presented in following form:

$$k_a = \omega\left(1+\frac{\chi_0}{2}\right) + \frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda'_{\mathbf{g}a}-\frac{\omega\beta}{2}\right), \quad k_c = \omega\left(1+\frac{\chi_0}{2}\right) + \frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda'_{\mathbf{g}c}-\frac{\omega\beta}{2}\right),$$

where

$$\lambda'_{\mathbf{g}a} = \lambda^*_{\mathbf{g}}-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi_a}{2}\right), \quad \lambda'_{\mathbf{g}c} = \lambda^*_{\mathbf{g}}-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi_c}{2}\right),$$

$$\lambda^*_{\mathbf{g}} = \frac{\omega\beta}{2} + \frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda^*_0, \quad \lambda^*_0 = \omega\left(\frac{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi_0}{2}\right), \quad \beta = \frac{1}{\omega^2}(k_{\mathbf{g}}^2-k^2)-\chi_0\left(1-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\right)$$

$\gamma_0 = \cos\Phi_0$, $\gamma_{\mathbf{g}} = \cos\Phi_{\mathbf{g}}$, Φ_0 is the angle between the wave vector $\mathbf{k}$ of the