Precision Engineering 38 (2014) 956–963

Contents lists available at ScienceDirect

Precision Engineering

journal homepage: www.elsevier.com/locate/precision

Technical note

Modeling and characterization of piezoelectric cantilever in fluids at different temperatures

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ARTICLE INFO

Article history:
Received 31 May 2013
Received in revised form 3 April 2014
Accepted 9 April 2014
Available online 2 May 2014

Keywords:
Piezoelectric
Cantilever
Fluid
Temperature

ABSTRACT

This paper introduces measurement and modeling of a piezoelectric beam used as a sensor in different types of liquids. It is immersed in different fluids at temperatures increasing from 20 °C to 50 °C. The working principle is based on detecting different resonance frequencies of the cantilever in different solutions. The oscillation of piezoelectric beam is measured using a vector network analyzer. An electrical equivalent circuit derived from a resonator model is used to simulate the experimental data. These calculated circuit constants have been related to physical properties of liquids under test. The combination of these liquids which includes non-conducting and conducting solutions, exhibiting low and high viscosity covers a good range of common physical properties of fluids. Main focus of this research is to explore the capability of piezoelectric cantilever as a liquid sensor with the influence of temperature. The equivalent circuit model has been proved to be viable to fit experimental data in non-conducting solution but less effective in conducting solution.

1. Introduction

The use of quartz crystal microbalance (QCM) as liquid sensor is a widely understood technique. Researchers have been using QCM to monitor viscosity of various solutions [1]. However, the advent of miniature cantilever which appears to be a more versatile sensor has gained attention from many areas. Its capability in detecting presence of organic or inorganic molecules has made this miniature, simple and robust electromechanical beam as sensor for many applications in chemistry, environmental, physics, biomedical and atomic force microscopy (AFM) [2,3]. Microcantilever has therefore spurred much interest among researchers to utilize it as a liquid sensor.

Different approaches have been used to measure the properties of liquid by using different designs of microcantilever. There have been attempts to modify off-the-shelf piezoelectric cantilever before being used as sensor. There is report on adding a tip at the unclamped end of a commercially available cantilever with only the tip immersing in liquid. Another method developed is by making asymmetric anchor for the piezoelectric cantilever at the clamped end. Theoretical model was used to explain the mechanism and properties of solutions were deduced from the governing equations [4,5]. Attempts to fabricate custom designed micromachined cantilever have also been presented to measure viscosity of complex organic liquid [6]. Studies had related viscosity to resonant frequencies change of the cantilever. Experiments involving Newtonian fluids [7] and polymer samples [8] have been demonstrated to explain the relationship between frequency response of sensor and liquid properties. Equations to determine the density and viscosity of unknown fluids have also been developed from rheological measurement of fluids using cantilever [9]. Resonant frequency dependency on fluid density was shown in all these work.

It is often more convenient to use equations or equivalent circuits to explain the analogy and relate measurement obtained to properties of measurands. Methods that have been demonstrated included modeling a clamped-clamped beam using Bernoulli–Euler equation and linearized Navier–Stokes equations for fluid's density and viscosity [10], examining the frequency response of piezoelectric cantilever in liquids at various excitation levels to determine viscosity and density of liquids [11], measuring density and viscosity of Newtonian liquids using complex parameters [12] and including the effect of longitudinal wave generated by a piezoelectric quartz crystal in the liquid in measurement [13]. Circuit model of resonator based on Butteworth–Van Dyke (BVD) model has been presented [14–17]. In this model, properties of fluid are represented by lumped elements of simple electrical circuit. An electrode-separated piezoelectric sensor in fluids is also successfully modeled by using a modified BVD equivalent circuit. [18].

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http://dx.doi.org/10.1016/j.precisioneng.2014.04.001
0141-6359/

Another approach introduced is based on S-parameters an electromechanical model of a piezoelectric transducer which resembles BVD circuit model was developed [19,20].

In this paper, modeling and measurement on piezoelectric cantilever as a liquid sensor has been conducted at different temperature environments. To the best of our knowledge, it is the first time for this type of sensors being investigated at various temperatures. Equivalent circuits for the piezoelectric cantilever vibrating in air and different types of fluids at varies temperatures have been developed. Characterization of the device under different environments has also been presented. The paper is organized as follows: Section 2 presents theoretical models for the device in different liquids, Section 3 describes the experimental procedures and results, finally discussion and conclusions are shown in Sections 4 and 5.

2. Modeling

2.1. Modeling in air

So far, most of the research only focuses on fitting a theory model or circuit model into a range of liquids with known properties at constant temperature. This paper is focused on the capability of a simple cantilever as a sensor to detect unknown physical properties of different liquids with the effect from temperature variation. This is essential, as in reality many properties are temperature dependent while temperature is unlikely to be constant all the time.

Equivalent circuit used in this paper is based on the complex circuit model proposed by Sherrit et al. [21] as shown in Fig. 1. Compared to BVD equivalent circuit as depicted in many past researches, this model is much more versatile. The parameter constants can be determined by using a few methods. By measuring impedance, half power frequencies, $f_{+1/2}$ and $f_{-1/2}$, series frequency, f_s and parallel frequency, f_p, other parameters of the complex circuit model can be determined. Another viable approach is to use nonlinear least-squares fit to find the values of circuit constants. The complex circuit model can be converted to conventional BVD circuit easily by applying basic circuit theory [21]. The basis of the complex circuit model is the same as described in previous researches [14–20] in which a resonator is used but the way of obtaining the circuit constants is much simpler than BVD method. Hence, this model can still be used to represent a liquid-immersed cantilever in which the properties of liquid are unknown and the methods mentioned can all be used to determine the values of equivalent circuit constants. A conversion to BVD is required to acquire the properties of liquid which are all in real numbers.

In the circuit model, f_s is defined as frequency of $G(f)/f$ at maximum and f_p is the frequency at maximum of $fR(f)$ [22] where $G(f)$ and $R(f)$ are the conductance and resistance of the piezoelectric beam. The half-power frequencies $f^s_{-1/2}$ and $f^s_{+1/2}$ of f_s as well as $f^p_{-1/2}$ and $f^p_{+1/2}$ of f_p are determined to form a complex series and parallel frequencies [11,12] which are given as below.

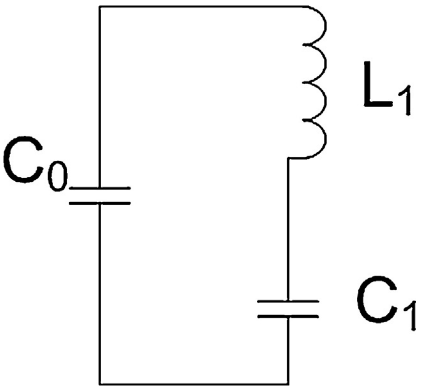

Fig. 1. Complex circuit model.

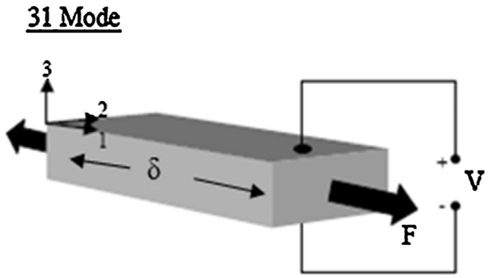

Fig. 2. Operation mode of cantilever.

$$\hat{f}_s = f_s\left[1 - i\frac{f^s_{-1/2} - f^s_{+1/2}}{f_s}\right]^{-(1/2)} \tag{1}$$

$$\hat{f}_p = f_p\left[1 - i\frac{f^p_{-1/2} - f^p_{+1/2}}{f_p}\right]^{-(1/2)} \tag{2}$$

where $\hat{f}_s$ and $\hat{f}_p$ are the complex series resonance and parallel resonance respectively. The impedance of such model is given as a function of ω [23]

$$Z = \frac{t/i\omega\varepsilon_{33}A}{[1 - k_{31}^2(1 - (\tan(\omega/4f_s)/(\omega/4f_s)))]} \tag{3}$$

Rearrange Eq. (3) yields

$$\varepsilon_{33} = \frac{t/i\omega ZA}{[1 - k_{31}^2(1 - (\tan\omega/(4f_s)/\omega/(4f_s)))]} \tag{4}$$

t is the thickness of beam, $A = wl$ is the area with w is the width and l is the length of beam. ε_{33} can be determined by fitting measured data points into Eq. (4). The electromechanical coupling constant K^2_{31} is given by

$$\frac{k_{31}^2}{1 - k_{31}^2} = \frac{\pi}{2}\frac{\hat{f}_p}{\hat{f}_s}\tan\frac{(\hat{f}_p - \hat{f}_s)}{\hat{f}_s} \tag{5}$$

With values of ε_{33} and K^2_{31} known, the impedance of model as a function of frequency can be determined. The material properties represented by C_0, C_1 and L_1 can be obtained using the following equations

$$C_1 = \frac{\varepsilon_{33}A/t}{1 - k_{31}^2}\frac{(\hat{f}_p - \hat{f}_s)}{\hat{f}_s} \tag{6}$$

$$C_0 = \frac{\varepsilon_{33}A/t}{1 - k_{31}^2} - C_1 \tag{7}$$

$$L_1 = \frac{1}{4\pi^2\hat{f}_s^2 C_1} \tag{8}$$

2.2. Modeling in liquid

Since the cantilever is dipped vertically into liquid, the largest force is exerted at the tip of beam as liquid pressure increases with depth. Therefore, the cantilever is strained across its length and operates in 31 mode as shown in Fig. 2. Identifying the operation mode of piezoelectric beam is essential to have an accurate analysis and modeling.

When the sensor is immersed in liquid, the same circuit model described in Section 2.1 applies but with the introduction of extra components to include the effect of liquid on the resonating circuit. A parallel capacitance is added to the original circuit and extra