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Quantum effects for particles channeling in a bent crystal

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ABSTRACT

Quantum mechanical theory for channeling of the relativistic charged particles in the bent crystals is considered in the paper. Quantum effects of under-barrier tunneling are essential when the radius of the curvature is closed to its critical value. In this case the wave functions of the quasi-stationary states corresponding to the particles captured in a channel are presented in the analytical form. The efficiency of channeling of the particles and their angular distribution at the exit crystal surface are calculated. Characteristic experimental parameters for observation the quantum effects are estimated.

1. Introduction

The first experimental observation of the charged particle channeling in a bent single crystal was reported in Ref. [1]. This phenomenon was theoretically predicted by Tsyganov in Ref. [2]. At present, the effect is considered as an perspective method in high energy physics to control the charged particle beams. A lot of experimental works were done in order to investigate accurately various aspects of the phenomenon both for positively [3–7,11] and negatively charged particles [8–10] with different energies. The length of dechanneling for high-energy protons in the bent crystal was measured with high precision [11]. In the papers [12,13] the undulator on the basis of the bent crystal was considered theoretically and was realized recently in [14].

The angle of the particle beam rotation is defined by the crystal length and its curvature radius R_{cr}. This value was estimated in dependence on the particle energy in Ref. [2]. All above mentioned experiments were operated with the crystals having the bent radius $R \gg R_{cr}$. In this case the theoretical simulation of the particle channeling can be fulfilled in the framework of the classical mechanics as it takes place for the channeling in the straight crystal.

The maximal angle of the particle beam rotation corresponds to the minimal possible crystal curvature radius $R \approx R_{cr}$. However, as it was shown in Ref. [15], the classical theory is not applicable in this case. Therefore, it is of interest to investigate in detail the particle motion in the bent crystals taking into account the factors that are not described within the framework of the classical theory of the phenomenon.

In this paper (see also in [15]) the quantum theory of the planar channeling of the relativistic particles in a bent single-crystal is built and the quantum effects are described at the crystal curvature radius in the range $R \approx R_{cr}$. It is shown that in this case the particle states in a bent crystal are changed substantially because of tunneling under the barrier created by the crystallographic planes. It leads to the change of the efficiency of capture in the channeling modes and the angular distribution of the particles at the exit surface of the bent crystal. All numerical calculations are fulfilled for the proton channeling in the Si crystals but the analysis is valid also for the negatively charged particles.

2. Stationary states of the particle channeled by the bent crystal.

Let us consider the equation, which follows from the Dirac equation and determines the stationary states of the relativistic particle with energy E and mass $m \ll E$ in a crystal if the small spin effects are not taken into account [15,16] (the natural system of units with $\hbar = c = 1$ is used):

$$\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} + E^2 - m^2 - 2EV(\vec{r})\right\}\Psi(\vec{r}) = 0. \quad (1)$$

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The average potential of the crystallographic planes in the bent crystal has cylindrical symmetry [3] and is described by the following function

$$V(r) = \sum_{n=-n_1}^{n_1} V_1(r - R - nd).$$

Here $V_1(r-R-nd)$ is the potential of a single plane; R is the curvature radius of the central channel, $2n_1$ is the number of crystallographic planes in the direction perpendicular to the bend; $d \ll R$ is the interplane distance.

The variables in Eq. (1) can be separated in the cylindrical coordinate system due to the potential symmetry:

$$\Psi(\vec{r}) = \frac{u(r)}{\sqrt{r}} \exp[i(l\varphi + p_z z)]; \quad l = 0, \pm 1, \pm 2, \ldots.$$

To solve this equation it is convenient to introduce relative radial variable $x = r - R; |x| < n_1 d \ll R$ and use the condition $|V(r)| \ll E$. Then the equation for the function $u(r)$ becomes similar to Schrodinger equation for the particle transverse motion in the case of planar channeling [17]

$$\left\{ -\frac{d^2}{dx^2} + 2E_0 V_{eff}(x) \right\} u(x) = \varepsilon u(x).$$

The total energy eigenvalue in Eq. (1) is determined by the quantum numbers l, p_z and the energy of radial motion ε in the effective potential $V_{eff}(x)$:

$$E \approx E_0 + \varepsilon; \quad E_0^2 = m^2 + p_z^2 + \frac{l^2 - 1/4}{R^2};$$
$$V_{eff}(x) = V(x) + \frac{p_0^2}{E_0 R} x = V(x) + \frac{p_0 v}{R} x; \quad p_0 = \sqrt{E_0^2 - m^2 - p_z^2}, \qquad (2)$$

with $v = p_0/E_0$ as the particle velocity.

From the classical point of view the bound state of the particle in a channel is appeared when the potential energy $V_{eff}(x)$ has a minimum in the range $-d/2 \leqslant x \leqslant d/2$. When considering Eq. (2) this condition takes the following form:

$$-|V'(x)|_{max} + \frac{p_0 v}{R} \leqslant 0. \qquad (3)$$

The crystallographic plane potential is the monotonically increasing function for $x \to \pm d/2$, therefore the condition (3) is equivalent to the following inequality for the average radius of the crystal curvature:

$$R \geqslant \frac{p_0 v}{|V'(x)|_{max}} \equiv R_{cr}, \qquad (4)$$

which coincides with expression obtained by Tsyganov [2].

In order to illustrate the further results quantitatively let us choose Si crystal bent along the planes (1 1 0) ($d = 1.92$ Å) as an example. In this case the potential of a single plane is well approximated by the Peschl-Teller potential [17]:

$$V_1(x) = a_{PT} \tanh^2(x/b_{PT}), \qquad (5)$$

with the parameters $a_{PT} = 23$ eV, $b_{PT} = 0.145d$ and $V'_{1max} = 6.37$ GeV/cm.

Remind, that the channeling potential have already took into account averaging of the microscopic particle-crystal potential over the atomic thermal vibrations [17]. Deviations from this potential conditioned by the particle-phonon interaction lead to an incoherent scattering and define one of the contributions to the dechanneling processes.

If one considers the channeling of protons with the energy $E = 70$ GeV, for which one of the first experiments with a bent crystal was carried out [1], $R_{cr} \approx 11.01$ cm. Fig. 1 shows the potential $V_{eff}(x)$, obtained by (5) and (2) with $E_0 = 70$ GeV, and the curvature radius $R = 12.01$ cm.

However, in a quantum theory the condition (4) is not sufficient to ensure that the particle could be captured in channel and change the velocity direction at a large angle. It happens because of the possibility of the particle tunneling under potential barrier (between the points x_1 and x_0 in Fig. 1). In the result it passes to the continuous spectrum state corresponding to a direct motion of the particle. Lifetime of the particle in a bent channel, and consequently, the angle of the particle rotation depends on the concrete form of the potential.

Note that one can consider the quantum effect of under-barrier tunneling as an additional mechanism of the particle dechanneling along with the known classical processes [3,11]. At the considered particle energy such quantum effects are negligible in the case of planar channeling in the straight crystal. But the barrier penetrability grows significantly when the curvature radius of the crystal is close to its critical value. Fortunately in this case all calculations can be conducted analytically for arbitrary $V(x)$, because $V_{eff}(x)$ can be taken into account in the harmonic approximation. With the above parameters $R_{cr} \approx 11.01$ cm and we will choose the crystal bend radius close to this value, for example, $R = 12.01$ cm (Fig. 1). In this case, the potential of a bent channel near barrier can be approximately written in the following form:

$$V_{eff}(x) \approx \begin{cases} V_1 = \frac{1}{2} V''(x_0)(x - x_0)^2; & x' < x < x_0; \\ V_2 = \Delta V - \frac{1}{2} |V''(x_1)| (x - x_1)^2; & x < x'. \end{cases} \qquad (6)$$

Here the point x' is determined from the matching condition $V_1(x') = V_2(x')$ and $\Delta V = V_{max}(x_1) - V_{min}(x_0)$.

In order to avoid misunderstanding it should be stressed that the harmonic approximation (6) for $V_{eff}(x)$ differs essentially from that one for channeling potential in the straight crystal. The latter one is used usually for x near the minimum of $V(x)$. On the contrary the points x_0, x_1 corresponding to the minimal value of $V_{eff}(x)$ are disposed near the point of inflection x_d for $V(x) : V'(x_d)_{max}$, $V''(x_d) = 0$; $|x_0 - x_d| \sim (R - R_{cr})$ (Fig. 1). All these points are close to the atomic planes and correspond to the potential maximum for the positively charged particles and to the potential minimum for the negatively charged particles. Interpolation (6) does not depend on the detailed form of the channeling potential on the whole interval but only on the values $V''(x_0), V''(x_1)$. It can be verified that these values change unessentially for all commonly used model channeling potential and for both particle charges [17].

For the potential (6) the quasi-stationary energy levels for the particle in the bent channel can be approximately calculated by means of the formula:

$$\varepsilon_k \approx \omega\left(k + \frac{1}{2}\right) - i\Gamma_k/2 \equiv \varepsilon_k^{(0)} - i\Gamma_k/2; \omega = \left[\frac{V''(x_0)}{E_0}\right]^{1/2}. \qquad (7)$$

The width Γ_k of the level can be found by using the quasi-classical expression for the penetration coefficient of the potential barrier [18]

$$\Gamma_k = A\omega \exp\left[-2\int_b^a \sqrt{2E_0(V_{eff}(x) - \varepsilon_k^{(0)})}dx\right], \qquad (8)$$

where $A \approx 1$ is pre-exponentials and the two turning points a and b are defined by the expression:

$$a, b = x_1 \pm \sqrt{\frac{2\Delta V - (2k+1)\sqrt{\frac{V''(x_0)}{E_0}}}{|V''(x_1)|}}.$$

Maximal number of the levels corresponding to the particle bound states is defined by the condition:

$$k_{max} < \frac{\Delta V}{\omega} - \frac{1}{2}$$