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Elastic photonuclear cross sections for bremsstrahlung from relativistic ions

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ABSTRACT

In this paper, we provide a procedure to calculate the bremsstrahlung spectrum for virtually any relativistic bare ion with charge $6e$ or beyond, $Z \geqslant 6$, in ultraperipheral collisions with target nuclei. We apply the Weizsäcker–Williams method of virtual quanta to model the effect of the distribution of nuclear constituents on the interaction of the ion with the radiation target. This leads to a bremsstrahlung spectrum peaking at 2γ times the energy of the giant dipole resonance (γ is the projectile energy in units of its rest energy). A central ingredient in the calculation is the cross section for elastic scattering of photons on the ion. This is only available in the literature for a few selected nuclei and, usually, only in a rather restricted parameter range. Hence we develop a procedure applicable for all $Z \geqslant 6$ to estimate the elastic scattering. The elastic cross section is obtained at low to moderate photon energies, somewhat beyond the giant dipole resonance, by means of the optical theorem, a dispersion relation, and data on the total absorption cross section. The cross section is continued at higher energies by invoking depletion due to loss of coherence in the scattering. Our procedure is intended for any ion where absorption data is available and for moderate to high energies, $\gamma \gtrsim 10$.

1. Introduction

When a charged particle penetrates a material, deflection in the electromagnetic field of target atoms causes emission of photons. We shall label this emission *bremsstrahlung* provided the original particle is left intact. For light particles like the electron, bremsstrahlung is the dominant source of energy loss when traveling at sufficiently high relativistic energies. While the process is also relevant for other light particles like muons [1], it's less so for protons [2]. Bremsstrahlung from heavy charged particles penetrating matter at high energies has never been measured. If the projectile is treated as a point charge, classical and quantal calculations indicate that bremsstrahlung should be a major energy-loss channel for highly relativistic bare ions and, as for electrons, ultimately the dominant one. However, when due account is taken for the internal structure of the nucleus and the latter is required to remain intact, which restricts to ultraperipheral collisions with target nuclei, the emission ends up confined to relatively soft quanta with the result that bremsstrahlung never adds substantially to the energy loss [3,4].

We have previously determined the bremsstrahlung spectrum for bare lead ions penetrating a lead target at relativistic energies (typically $\gamma = 170$ and beyond). Our calculation was based on the Weizsäcker–Williams method of virtual quanta. There are four contributions, the main contribution derives from scattering of virtual photons of the target nucleus on the incoming ion. The scattering is determined in the rest frame of the projectile and the main bremsstrahlung component is obtained by subsequent transformation to the target frame. Obviously, a central ingredient is the photo-nuclear scattering cross section. For the case of lead experimental data for elastic photon scattering are available; these data were used to construct a relatively accurate fit which served as input in the calculation of bremsstrahlung for a lead ion [3]. The nature of the photon-nucleus interaction is reflected in the structure of the bremsstrahlung cross section, which shows a significant peak at, roughly, 25γ MeV, enhanced above the results for a point-like source of the same charge and mass.

Computation of the bremsstrahlung spectrum for heavy ions other than lead is complicated by the fact that data for elastic photon scattering is generally unavailable. The elastic cross section therefore has to be estimated by other means. For such cases, we model the elastic cross section from the total absorption cross section by application of the optical theorem and a dispersion relation at energies up to and somewhat beyond the giant dipole resonance and include depletion at higher energies based on the underlying physics. We adjust so as to obtain a single closed procedure

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applicable for all nuclei by requiring reproduction at an acceptable level of elastic scattering data available for C, O, and Pb.

While the theoretical results for lead ions have awaited experimental data to check against, other relevant experiments may soon be underway. Starting in 2015, Ar, Xe and Pb beams will be available at the Super Proton Synchrotron at CERN for fixed-target experiments. We shall therefore apply our procedure to the case of argon.

As discussed below, the relative widths of the expected radiation peaks give indirect information on the Giant Dipole Resonance of the projectile. These widths thus inform about the symmetries of the nucleus – if it is spherical or not, for instance. The level of accuracy with which this information can be obtained by such an indirect method is not competitive in the case of stable elements, but in the case of bremsstrahlung emission it may yield information even for nuclei with lifetimes down to the picosecond regime.

The reader should note that the nuclear community applies the term bremsstrahlung much broader than we do here. It is used for photon emission even in energetic central collisions where both collision partners disintegrate. The more restrictive use of the term here follows from our perspective, viz. the slowing down of charged particles.

2. Bremsstrahlung in ultraperipheral collisions

The classical radiation cross section for a pointlike particle serves as a reference for our bremsstrahlung results. For a pointlike bare relativistic ion with charge Ze, nucleon number A, and mass $M = AM_u$ penetrating a target of atomic number Z_t at energy E, this cross section is given as [5]

$$\frac{d\chi}{d\hbar\omega} = \frac{16}{3}\frac{Z_t^2 Z^4}{A^2}\alpha r_u^2 L, \tag{1}$$

where $\alpha \equiv e^2/\hbar c^2$ is the fine-structure constant, $r_u \equiv e^2/M_u c^2$ the classical nucleon radius, and M_u the atomic mass unit. Note that χ carries the dimension of energy times area; a count spectrum is obtained by dividing by the photon energy $\hbar\omega$. The factor L appearing in Eq. (1) is given by

$$L \approx \ln\left(\frac{233M}{Z_t^{1/3}m}\right) - \frac{1}{2}\left[\ln\left(1+r^2\right) - \frac{1}{1+r^{-2}}\right], \quad r = \frac{96\hbar\omega}{\gamma\gamma_- Z_t^{1/3}mc^2}, \tag{2}$$

where m denotes the electron mass, $\gamma \equiv E/Mc^2$, and $\gamma_- \equiv (E - \hbar\omega)/Mc^2$. It is essentially the logarithm of the ratio of the effective maximum and minimum momentum transfers to the scattering center. The logarithmic factor L represents the only energy-dependent part of the radiation cross section (1) which thereby is almost constant up to the primary energy E (neglecting quantum recoil).

To obtain the bremsstrahlung emitted by a composite nucleus penetrating a target at relativistic energy, the Weizsäcker–Williams method of virtual quanta was applied in [3]. The nuclear structure plays an essential role in the photon emission and leads to a significant energy-dependence of the cross section. In the Weizsäcker–Williams approach, the bremsstrahlung stems from a combination of four sources: scattering of virtual photons of the target nucleus on the projectile, scattering of virtual photons of the target electrons on the projectile, scattering of virtual photons of the projectile on the target nucleus, and scattering of the virtual photons of the projectile on target electrons [4]. The main component is due to scattering of virtual photons from target nuclei on the projectile. When these scatter at large angles they undergo a Lorentz boost that increases their energy by $\approx 2\gamma$ easily resulting in the emission at GeV energies for virtual photons incident at 10–20 MeV on relativistic projectiles with γ of the order of 10^2.

For the main bremsstrahlung component, the scattering of the virtual photons of the target nucleus on the projectile is considered in the rest frame of the latter, where variables are denoted by primes. By multiplying the elastic photon scattering cross section differential in scattering angle, $d\sigma/d\Omega'$, with the spectrum of virtual photons differential in photon energy, $dI'/d\hbar\omega'$, we obtain the doubly-differential radiation cross section

$$\frac{d^2\chi'}{d\hbar\omega' d\Omega'} = \frac{d\sigma}{d\Omega'}\frac{dI'}{d\hbar\omega'}. \tag{3}$$

Explicit expressions for the Weizsäcker–Williams photon intensity, $dI'/d\hbar\omega'$, are given in Refs. [3,4]. We shall apply the intensity pertaining to an exponentially screened Coulomb potential of the target nuclei. Determination of the elastic cross section $d\sigma/d\Omega'$ is the major issue of this paper; it is discussed in Section 3 below.

To obtain a measurable bremsstrahlung yield, the radiation cross Section (3) has to be transformed to the laboratory frame (rest frame of the target). For this, use is made of the invariance relation [5,3]

$$\frac{1}{\omega^2}\frac{d^2\chi}{d\hbar\omega d\Omega} = \frac{1}{\omega'^2}\frac{d^2\chi'}{d\hbar\omega' d\Omega'}. \tag{4}$$

We obtain the bremsstrahlung spectrum in the laboratory, differential in energy and solid angle, by multiplication of Eq. (4) by ω^2 and by expressing the photon energy and scattering angle in the projectile rest frame, appearing on the right-hand-side of the equation, in terms of their laboratory equivalents. From the relativistic Doppler (or Lorentz) transformation we have

$$\omega' = \gamma\omega(1 - \beta\cos\theta) \tag{5}$$

and

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}, \tag{6}$$

where θ is the angle between the direction of photon emission and the direction of the projectile and $\beta = v/c$ is the projectile speed relative to that of light.

Eqs. (3)–(6) summarize the necessary ingredients to calculate the bremsstrahlung spectrum; additional information as well as useful and quite accurate approximations relying on high values of γ may be found in Ref. [3]. Fig. 1 displays the result for the main

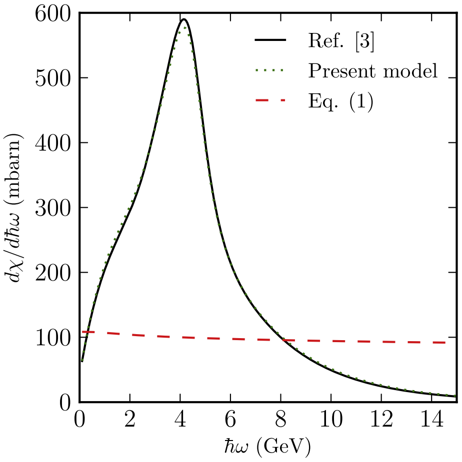

Fig. 1. Bremsstrahlung spectrum for a bare lead nucleus penetrating a lead target at a γ-value of 170. The solid black line is adapted from [3], the green dotted line shows the result of the present approach, Section 3.2, while the red dashed line shows the reference cross section (1). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)