Nuclear Instruments and Methods in Physics Research B 362 (2015) 19–24

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Channeling of fast positrons on a crystal surface in a uniform magnetic field

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ARTICLE INFO

Article history:
Received 31 July 2015
Received in revised form 28 August 2015
Accepted 29 August 2015

Keywords:
Crystal
Charged particle
Channeling
Spectrum
Polarization

ABSTRACT

The conditions of channeling of relativistic positively charged particles near the surface of a crystal in the presence of a magnetic field parallel to the surface are studied. It is shown that the emission spectrum of a single particle is discrete. The frequency of the first harmonic and the number of harmonics, making a significant contribution to the spectrum depend strongly on the initial coordinates and velocities of the particles. When averaged over the initial coordinate of the particle, the spectrum becomes continuous. The spectral range in which the radiation of parallel beam of particles is generated, does not depend on the magnetic field and is determined only by the energy of the particles and the averaged potential of the surface atomic layer. Radiation is concentrated in a narrow cone in the direction of the average velocity of the particles and polarized essentially in the plane orthogonal to the crystal surface.

1. Introduction

Motion of charged particles in a crystal along the channels formed by parallel rows of atoms or atomic planes has been predicted by American physicists Robinson and Owen in 1961 [1] and first observed experimentally in 1963–1965. Channeling of the charged particles is an effective method for studying the structure and properties of crystals and some nuclear phenomena. New sources of X-ray and gamma radiation, consisting of the particle accelerators and precisely oriented mono-crystals are created on the basis of the channeling phenomenon. Currently, there is interest in developing a radiation source using the periodically deformed crystals. See, for example, the recent papers [2,3]. In this case, the channeled positrons are bound to the bent atomic crystal planes, and emit monochromatic radiation of a frequency which depends on the particles energy and the period of the crystal plate bending. Such a "crystalline undulator" was first proposed in [4,5], where the deformation of the crystal was proposed to be performed by an ultrasonic wave. There are also a number of schemes of multi-crystal undulators in which the particle is deflected successively in the opposite directions at passing through a series of mutually oriented ultrathin crystals. See, for example, the brief review [6] and references therein. The radiation generated in the crystal undulators can be harder than one can get in a magnetic undulator at the same energy of the positrons, since the period of the deformed crystal may be much smaller than the period of undulator magnetic field.

A "magneto-crystalline undulator" in which generation of electromagnetic radiation by means of charged particles moving near the flat surface of a crystal in a magnetic field parallel to the surface was proposed in [7]. The positively charged particles at grazing incidence to the surface of the crystal are reflected from the surface and again returning by the magnetic field, if the field is orthogonal to the average velocity of the particles. The quantum energy levels of the particle transverse motion at the surface channeling in a magnetic field, and the possible frequency of radiation were also found in [8].

Interaction of the particles with the crystal surface in the presence of a magnetic field can be observed when the crystal is mounted on the beam path in the chamber of a circular accelerator. The dynamics of the beam in an accelerator at its interaction with a radiator which is thin in profile and extended along the beam, in case of grazing incidence geometry is not investigated in accelerator physics.

An effect of asymmetric generation of the X-ray radiation produced at grazing interaction of 33 MeV electrons with 50 mm Si plate in a magnetic field have recently been observed in experiments reported in [9]. The crystal plate of length 4 mm

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http://dx.doi.org/10.1016/j.nimb.2015.08.077

was placed in a goniometer within the betatron chamber parallel to the electron beam. The evolution of angular patterns formed by the X-rays generated in the Si plate, when changing the plate orientation relative to the electron beam, was studied. The experimental results demonstrated preferential generation of X-rays on the Si plate surface, which was external with respect to the centre of the accelerator. At grazing incidence of electrons on this surface the radiation was emitted along the Si plate surface in the cone, which was several times narrower than the cone of ordinary bremsstrahlung emitted along the electron beam direction.

The determining factor in the observed effect is the influence of the accelerator magnetic field on interaction of electrons with the plate. The grazing incidence of the electrons on the inner or outer surface with respect to the accelerator centre are not equivalent, because of differences in the trajectories of electrons scattered on the Si plate. At grazing interaction of electrons with the surface facing towards the centre of accelerator, the generation of near-surface radiation was not observed.

In this paper we investigate the spectral and angular distribution of the radiation energy at the surface channeling of fast positrons. It is assumed that the crystal is placed in a uniform magnetic field parallel to the crystal surface. The initial velocity of the particle is directed so that the Lorentz force exerted by the magnetic field is directed to the surface of the crystal. The result of the action of two competing forces – the force caused by the magnetic field and the repulsive force of the crystallographic plane, is that the particle oscillates in the vicinity of the surface.

2. The effective potential energy

Let us direct the axis Z along the vector of the magnetic field, and the axis Y be orthogonal to the surface of the crystal directed outward (Fig. 1). The axis X lies in the crystal surface so that the vector $\boldsymbol{v}$ of the particle initial velocity is lying in the plane XY. The crystallographic plane forming the surface of the crystal has the coordinates $y = 0$. We define the potential of the electric field of the crystallographic plane according to [10]

$$U(y) = \frac{U_0}{a}\left[\left(y^2 + a^2\right)^{1/2} - y\right], \tag{1}$$

where U_0 is the averaged potential of atomic plane, and a is the screening radius[1]. We consider an ultra-relativistic particle, hence, we neglect the interaction of the particle with its image in the crystal.

The electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields in the halfspace $y > 0$ can be written as

$$\boldsymbol{H} = (0, 0, H), \quad \boldsymbol{E} = (0, E(y), 0),$$

$$E(y) = -\frac{dU(y)}{dy} = -\frac{U_0}{a}\left(\frac{y}{\sqrt{y^2 + a^2}} - 1\right), \tag{2}$$

where H is a constant. Relativistic equations of motion of a particle in this field have the form

$$\frac{dp_x}{dt} = \frac{e}{c}\dot{y}H, \tag{3}$$

$$\frac{dp_y}{dt} = eE(y) - \frac{e}{c}\dot{x}H. \tag{4}$$

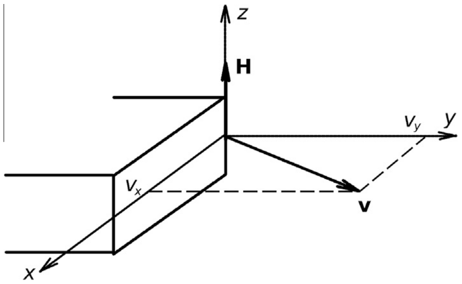

Fig. 1. The coordinate system.

Here p_x, p_y are projections of the particle momentum on the coordinate axes:

$$p_i = \gamma m_0 \dot{x}_i, \quad x_i = x, y,$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta^2 = \frac{\dot{x}^2 + \dot{y}^2}{c^2},$$

m_0 and e are the mass and the charge of the particle, c is the speed of light.

Relativistic factor γ in Eqs. (3) and (4) can be written as

$$\gamma^{-1} = \sqrt{1 - [\beta_{0x} + \delta\beta_x(t)]^2 - [\beta_y(t)]^2},$$

where β_{0x} is the projection of initial velocity on the axis X, $\beta_y(t)$ is the projection of the velocity on the axis Y, $\delta\beta_x(t)$ is the varying part of projection of the velocity on the axis X (all velocities in terms of the speed of light). As the particle is ultra-relativistic one ($\gamma \gg 1$), the particle velocity along the X axis is much greater than the velocity along the Y axis. More precisely, we assume that the conditions

$$\beta_y \ll \sqrt{1 - \beta_{0x}^2},$$
$$\delta\beta_x(t) \ll 1 - \beta_{0x}^2 \tag{5}$$

are fulfilled. Then the value of γ can be considered as constant and taken equal to $\gamma = \left(1 - \beta_{0x}^2\right)^{-1/2}$. The initial conditions under which the inequalities (5) hold, we will find after solution of the equations of motion.

Integration of the Eqs. (3) and (4) gives

$$\dot{x} = \omega y + V_1, \tag{6}$$

$$\begin{aligned} \frac{m\dot{y}^2}{2} &= \mathcal{E}_0 - \frac{eU_0}{a}\left(\sqrt{y^2 + a^2} - y\right) \\ &\quad - m\omega\left(\frac{\omega y^2}{2} + V_1 y\right). \end{aligned} \tag{7}$$

Here $m = m_0\gamma$, $\omega = eH/mc$, $\mathcal{E}_0$ and $V_1 = v_{x0} - \omega y_0$ are constants of integration. The last equation describes the one-dimensional motion of a particle in direction of the Y axis in the field with effective potential energy

$$U_{ef} = \frac{eU_0}{a}\left(\sqrt{y^2 + a^2} - y\right) + m\omega\left(\frac{\omega y^2}{2} + V_1 y\right).$$

The integral of motion $\mathcal{E}_0$ plays the role of "transverse" energy of the particle. It is convenient to introduce the dimensionless quantities

$$V = \frac{U_{ef}}{eU_0}, \quad \chi = \frac{y}{a}, \quad \eta = \frac{aH}{U_0} = \frac{amc^2}{ReU_0},$$

where $R = c/\omega$ is a parameter that determines the strength of the magnetic field. It is approximately equal to the particle orbit radius in this field. The parameter η is the ratio of the force acting on the particle by the magnetic field to the electrostatic repulsion force of the crystallographic plane. The dimensionless potential V with this notations takes the form

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[1] In accordance with [10], the screening radius can be set equal to $a = 0.8853\sqrt{3}a_0 Z^{-1/3}$, where a_0 is the Bohr orbit radius, Z is the atomic number.