Nuclear Instruments and Methods in Physics Research B 355 (2015) 110–113

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Combined effect in coherent bremsstrahlung of channeled electrons

Yu.P. Kunashenko *

National Research Tomsk Polytechnic University, Russian Federation
Tomsk State Pedagogical University, Russian Federation

ARTICLE INFO

Article history:
Received 30 November 2014
Received in revised form 26 March 2015
Accepted 26 March 2015
Available online 27 April 2015

Keywords:
Coherent bremsstrahlung
Channeled electron
Combined effect
Virtual photon method

ABSTRACT

Theory of coherent bremsstrahlung from axially channeled electrons in a crystal is developed in the framework of the virtual photons method. It is shown that electron channeling results in both splitting and appearing an additional structure of coherent peaks in axial coherent bremsstrahlung spectrum.

1. Introduction

In this work we study coherent bremsstrahlung from channeled electrons in a crystal.

Many physical processes, accompanying a passage of the high energy particles at small angles with respect to the crystal axes or planes, differ from those in an amorphous target. In fact, in this case coherent effects occur (see, e.g. [1–5]). The coherent effect in the interaction of charged particle with a crystal arises due to periodical arrangement of the crystal atoms, when the interaction amplitudes of the particle with individual atoms are summing up in the phase. As a result of this summing, an interferential multiplier appears in the cross-section. It leads to appearance of coherent peaks when the transferred momentum coincides with one of the reciprocal lattice vectors. This happens at selected parameters (beam energy, angles of incidence into a crystal with respect to crystallographic axes or planes).

Recently some interest in literature to the coherent bremsstrahlung from neutrons [6,7] was appeared.

The channeling phenomenon takes place when relativistic charged particle passes through a crystal at small angle with respect to the crystal axis or plane, and when the crystal thickness is rather large so that the Born approximation or many-wave diffraction approach do not work. In this case, a charged particle interacts with averaged (continuous) potential of the crystal axis or plane and one can neglect the periodic part of the crystal potential [3–5,8].

The coherent scattering or channeling of relativistic charged particles in a crystal leads to emission of coherent bremsstrahlung (CB) [1–5] and channeling leads to channeling radiation [3–5,8–10].

The CB and channeled radiation typically are considered as different kind of radiation from charged particles in a crystal. But it is possible to prepare especially condition when both channeling and coherent process can occur simultaneously. In this case a combined effect arises. Combined effect manifests itself in the change the energy spectrum of CB. For the first time combined effect was experimentally recorded at Tomsk synchrotron [11]. But, the accurate theory of combined effect still not exists.

2. Cross-section of the photon scattering by a channeled electron

In the reference frame related to the channeled electron passing through the crystal the electric field of an atom is perpendicular to its magnetic field. According to the Lorentz transformations, the fields are compressed in the longitudinal direction of the particle motion. Therefore, in a moving reference frame the electromagnetic field of the atoms of crystal is similar to the fields of a plane wave (if channeled electron relativistic factor $\gamma > 1$). Consequently, the electromagnetic field of the crystal can be replaced by a flux of virtual photons. Further, bremsstrahlung can be described in terms of the virtual photons scattering by channeled electron [1,6,7,12].

* Address: National Research Tomsk Polytechnic University, 634050, Tomsk, Lenin Avenue, 30, Russian Federation.
E-mail address: kunashenko@tpu.ru

http://dx.doi.org/10.1016/j.nimb.2015.03.076
0168-583X/

That is why we start our consideration with calculation of the cross – section of photons scattering by channeled electrons.

In the moving reference frame one can use nonrelativistic approach to the scattering process. The channeled electron wave function has a form

$$\Psi_{i(f)}(\mathbf{r}) = U_{i(f)}(\boldsymbol{\rho})\ \exp(i\mathbf{k}_{\|i(f)}\mathbf{r}_{\|}),$$

here $U_{i(f)}(\boldsymbol{\rho})$ is the channeled electron transverse wave function. In the moving coordinate system these functions are satisfied to the Schrödinger equation

$$\left[\varepsilon_{i(f)} - \frac{1}{2m}\hat{p}_{\perp}^2 + \gamma V(\boldsymbol{\rho})\right] U_{i(f)}(\boldsymbol{\rho}) = 0. \tag{1}$$

Here $\varepsilon_{i(f)}$ is the energy of electron transverse motion in the initial (final) state, γ is the electron relativistic factor in the laboratory coordinate frame, $V(\boldsymbol{\rho})$ is the continuous potential of crystal axis, $\hat{p}_{\perp}$ is the transverse momentum operator, m is the electron rest mass. The term $\exp(i\mathbf{k}_{\|i(f)}\mathbf{r}_{\|})$ describes the longitudinal electron free motion along crystal axis, with the longitudinal electron wave vector $\mathbf{k}_{\|i(f)}$.

We chose the continuous axial potential for electrons in the form [3]

$$V(\rho) = \frac{V_0}{\rho} \quad /\rho \neq 0/. \tag{2}$$

Here V_0 is the depth of the potential well. The transverse wave functions corresponding to this potential are well known (see for example [13]).

$$\begin{aligned} U_{nl}(\boldsymbol{\rho}) &= U_{nl}(\rho)\frac{1}{\sqrt{2\pi}}e^{il\varphi} \\ &= \sqrt{\rho} \\ &\quad \times \frac{1}{\Gamma(2l+1)}\left(\frac{\Gamma(n+l+\frac{1}{2})}{\Gamma(n-l+\frac{1}{2})2l}\right)^{1/2}(2\alpha_n)^{3/2e^{-\alpha_n\rho}}(2\alpha_n\rho)^{l-\frac{1}{2}} \\ &\quad \times F\left(-n+l+\frac{1}{2};2l;\alpha_n\rho\right)\frac{1}{\sqrt{2\pi}}e^{il\varphi}. \end{aligned} \tag{3}$$

Here $\alpha_n = \sqrt{-2\varepsilon_n mc^2}/\hbar c$, $F(-n+l+\frac{1}{2};2l;\alpha_n\rho)$ is the confluent hypergeometric function (the Kummer function), n is the principal quantum number, l is the magnetic quantum number, and electron transverse energies are described by the formula

$$\varepsilon_n = -\frac{(\gamma V_0/\hbar c)^2 mc^2}{2(n+l+\frac{1}{2})}. \tag{4}$$

The cross-section of the photon scattering by a channeled electron is described in the second-order perturbation theory [6,7,14,15]:

$$\begin{aligned} d\sigma_{FI} = \frac{2\pi}{\hbar c}&\left|\sum_{pol}\sum_{n}\sum_{l=-(n-1)}^{(n-1)}\left\{\frac{(\mathbf{e}_2\mathbf{R}_{Fnl})(\mathbf{e}_1\mathbf{A}_{nlI})}{(E_i+\hbar\omega_1)-E_n} + \frac{(\mathbf{e}_1\mathbf{A}_{Fnl})(\mathbf{e}_2\mathbf{R}_{nlI})}{E_i-E_n-\hbar\omega_2}\right\}\right|^2 \\ &\times \delta[(E_i+\hbar\omega_1)-(E_f+\hbar\omega)]\frac{(\hbar\omega_2)^2 d\hbar\omega_2 d\Omega}{(2\pi\hbar c)^3}. \end{aligned} \tag{5}$$

Here $\sum_{pol}$ is the sum over the incident and scattered photon polarization vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, $\sum_n\sum_{l=-(n-1)}^{(n-1)}$ is the sum over intermediate channeled electron states, and $F=\{f,l_f\}$, $I=\{i,l_i\}$, $\hbar\omega_1$ is the energy of initial photon and $\hbar\omega_2$ is the energy of scattered photon. We introduce in Eq. (5) the following notations:

$$\mathbf{A}_{KN} = -\frac{e}{m}\int \Psi_K^*(\mathbf{r})\hat{\mathbf{p}}\sqrt{\frac{2\pi\hbar}{\omega_1}}\exp[i\mathbf{k}_1\mathbf{r}]\Psi_N(\mathbf{r})d\mathbf{r}, \tag{6}$$

the matrix element of the photon absorption ($K=\{k,l_k\}$, $N=\{n,l_n\}$) and

$$\mathbf{R}_{KN} - \frac{e}{m}\int \Psi_K^*(\mathbf{r})\hat{\mathbf{p}}\sqrt{\frac{2\pi\hbar}{\omega_2}}\exp[-i\mathbf{k}_2\mathbf{r}]\Psi_N(\mathbf{r})d\mathbf{r}, \tag{7}$$

the matrix element of photon emission from the channeled electron, here e is the electron charge.

In equations (6 and 7), $\mathbf{k}_1$ is the photon wave vector, $\mathbf{e}_1$ is the polarization vector and ω_1 is the frequency of the incident photon while $\mathbf{k}_2$, $\mathbf{e}_2$ ω_2 are the wave vector, polarization vector and frequency of the scattered photon, respectively, $\hat{\mathbf{p}}$ is the electron momentum operator and $\Psi_I(\mathbf{r})$ is the channeled electron wave function.

If the incident photon moves along the OZ axis then its wave vector is defined as

$$\boldsymbol{k} = \{0,0,k\}, \quad k = \omega/c,$$

while the wave vector of scattered photon can be written as

$$\mathbf{k}_2 = \{k_2\sin\Theta\cos\Phi, k_2\sin\Theta\sin\Phi, k_2\cos\Theta\}, \quad k_2 = \omega_2/c.$$

Here, Θ and Φ are the photon scattering angles. The scattering angles are given in the cylindrical coordinates.

After substitution of the wave functions Eqs. (6 and 7) into Eq. (5) and some algebra we obtain

$$\begin{aligned} d\sigma_{FI} = \frac{r_0^2}{m^2}\frac{\hbar\omega_2}{\hbar\omega_1}&\left|\sum_{pol}\sum_{n}\sum_{l=-(n-1)}^{(n-1)}\left\{\frac{(\mathbf{e}_2\mathbf{R}_{Fnl})(\mathbf{e}_1\mathbf{A}_{nlI})}{(E_i+\hbar\omega_1)-E_n} + \frac{(\mathbf{e}_1\mathbf{A}_{Fnl})(\mathbf{e}_2\mathbf{R}_{nlI})}{E_i-E_n-\hbar\omega}\right\}\right|^2 \\ &\times \delta[(\mathbf{k}_{\|i}+\mathbf{k}_{\|1})-(\mathbf{k}_{\|f}+\mathbf{k}_{\|2})]\delta[(E_i+\hbar\omega_1) \\ &-(E_f+\hbar\omega)]d\Omega d\hbar\omega_2. \end{aligned}$$

Here r_0 is the classical electron radius.

In the dipole approximation the vectors $\mathbf{R}_{KN}$ and $\mathbf{A}_{KN}$ ($K=\{k,l_k\}$, $N=\{n,l_n\}$) have the following components:

$$(R_{KN})_x = i\pi m\Omega_{KN}C_2\int_0^{\infty} U_K(\rho)U_N(\rho)\rho^2 d\rho Dx(l_k,l_n),$$

$$(R_{KN})_y = i\pi m\Omega_{KN}C_2\int_0^{\infty} U_K(\rho)U_N(\rho)\rho^2 d\rho Dy(l_k,l_n),$$

$$(R_{KN})_x = p_{\|k}C_2\int_0^{\infty} U_K(\rho)U_N(\rho)\rho^2 d\rho Dz(l_k,l_n),$$

$$(A_{KN})_x = i\pi m\Omega_{KN}C_1\int_0^{\infty} U_K(\rho)U_N(\rho)\rho^2 d\rho Dx(l_k,l_n),$$

$$(A_{KN})_y = i\pi m\Omega_{KN}C_1\int_0^{\infty} U_K(\rho)U_N(\rho)\rho^2 d\rho Dy(l_k,l_n),$$

$$(A_{KN})_x = 0.$$

Here we introduce the following notations: $\Omega_{KN} = \frac{\varepsilon_n-\varepsilon_k}{\hbar}$, $C_{1(2)} = -\frac{e}{m}\sqrt{\frac{2\pi\hbar}{\omega_{1(2)}}}$. The functions $U_K(\rho) = U_{k,l_k}(\rho)$ are defined according to the Eq. (3)and

$$\begin{aligned} Dx(l_k,l_n) &= \int_0^{2\pi}\exp[-i(l_k-l_n)\varphi]\sin\varphi d\varphi \\ &= i\pi[\delta(l_n-l_k,\ 1)-\delta(l_n-l_k,\ -1)], \end{aligned}$$

$$\begin{aligned} Dy(l_k,l_n) &= \int_0^{2\pi}\exp[-i(l_k-l_n)\varphi]\cos\varphi d\varphi \\ &= \pi[\delta(l_n-l_k,\ -1)+\delta(l_n-l_k,\ 1)], \end{aligned}$$