Nuclear Instruments and Methods in Physics Research B xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Parametric X-ray radiation from composite bunches

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A R T I C L E I N F O

Article history:
Received 30 November 2014
Received in revised form 19 April 2015
Accepted 19 April 2015
Available online xxxx

Keywords:
Parametric X-ray radiation
Beam diagnostics
Composite bunch
Attosecond bunches

A B S T R A C T

Theory of parametric X-ray radiation (PXR) is developed for the case of composite bunch consisting of two fractions of charged particles with different charges and distributions. We suggest PXR as an instrument for the composite bunch diagnostics, for example in case of ion beams in crystal, consisting of two fractures of different ions. Also, for atto-second electron bunches the characteristics of coherent PXR are discussed.

1. Introduction

Parametric X-ray radiation (PXR) occurring when charged particles move in a crystal was predicted theoretically by Ter-Mikaelian in 1972 [1], and then observed experimentally at the Tomsk Synchrotron Sirius in 1985 [2]. Since then PXR has been studied very well, both theoretically and experimentally (see [3–5]). However, up to now PXR is not used in practice owing to rather weak brightness. Nevertheless, PXR is intensive enough to be detected experimentally, and can serve as a source of information about bunches of charged particles.

2. Polarization currents and field of X-ray parametric radiation

Using the kinematic theory of scattering of X-rays, it is not hard to obtain an expression for the electric field intensity at a distance $\mathbf{r}$ from the scattering of the crystal [1]:

$$\mathbf{E}^r(\mathbf{r},\omega) = \frac{ieZ}{\varepsilon(\omega)}\frac{e^{ikr}}{r}\sum_{\mathbf{g}\neq 0}\chi_{\mathbf{g}} e^{-i(\mathbf{k}-\mathbf{g})\mathbf{R}} \times \left[\mathbf{k}\left[\mathbf{k}\left\{\frac{\mathbf{g}+\mathbf{v}\varepsilon(\omega)\frac{\omega}{c^2}}{(\mathbf{k}-\mathbf{g})^2-k^2}\delta(\omega-(\mathbf{k}-\mathbf{g})\mathbf{v})\right\}\right]\right], \quad (1)$$

where

$$k^2 = \varepsilon(\omega)\frac{\omega^2}{c^2}, \qquad \varepsilon(\omega) = 1-\frac{\omega_p^2}{\omega^2}, \qquad \omega_p \ll \omega, \quad (2)$$

$\mathbf{k} = \frac{\omega}{c}\sqrt{\varepsilon(\omega)}\mathbf{n} = \frac{\omega}{c}\sqrt{\varepsilon(\omega)}\frac{\mathbf{r}}{r}$ is the wave-vector of the radiation. Let us consider PXR from a composite bunch of ions passing a crystal, on the basis of the clear physically conception of polarization currents (see, for example, Chapter 4 in monograph [6]). For M sorts of charged particles of quantity N_i with different charges $Z_i,\ i = 1, 2, \ldots, M$ one can obtain:

$$\mathbf{E}^r(\mathbf{r},\omega) = \frac{ie}{\varepsilon(\omega)}\frac{e^{ikr}}{r}\sum_{\mathbf{g}\neq 0}\chi_{\mathbf{g}}\sum_{i=1}^{M} Z_i \sum_{p=1}^{N_i} e^{-i(\mathbf{k}-\mathbf{g})(\mathbf{R}+\Delta\mathbf{r}_i^p)} \times \left[\mathbf{k}\left[\mathbf{k}\left\{\frac{\mathbf{g}+\mathbf{v}\varepsilon(\omega)\frac{\omega}{c^2}}{(\mathbf{k}-\mathbf{g})^2-k^2}\delta(\omega-(\mathbf{k}-\mathbf{g})\mathbf{v})\right\}\right]\right] \quad (3)$$

3. Parametric X-ray radiation from composite bunches

The composite bunch is thought to be mixture of two bunches of charged particles with different distributions and different properties of the single particles they consist of, see Fig. 1.

The field of radiation from two particles is

$$\mathbf{E}^r(\mathbf{r},\omega) = Z_1\sum_{\mathbf{g}\neq 0}\chi_{\mathbf{g}} e^{-i\mathbf{h}(\mathbf{R}_1+\Delta\mathbf{r}_1)}\mathbf{A}_{\mathbf{g}} d_{\mathbf{g}} + Z_2\sum_{\mathbf{g}'\neq 0}\chi_{\mathbf{g}'} e^{-i\mathbf{h}'(\mathbf{R}_1+\Delta\mathbf{r}_2)}\mathbf{A}_{\mathbf{g}'} d_{\mathbf{g}'}, \quad (4)$$

where

$$\mathbf{A}_g = \left[\mathbf{k}\left[\mathbf{k}\left\{\frac{\mathbf{g}+\mathbf{v}\varepsilon(\omega)\frac{\omega}{c^2}}{(\mathbf{k}-\mathbf{g})^2-k^2}\right\}\right]\right] \quad (5)$$

$$d_g = \frac{ie}{\varepsilon(\omega)}\frac{e^{ikr}}{r}\delta(\omega-(\mathbf{k}-\mathbf{g})\mathbf{v}) \quad (6)$$

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http://dx.doi.org/10.1016/j.nimb.2015.04.043
0168-583X/

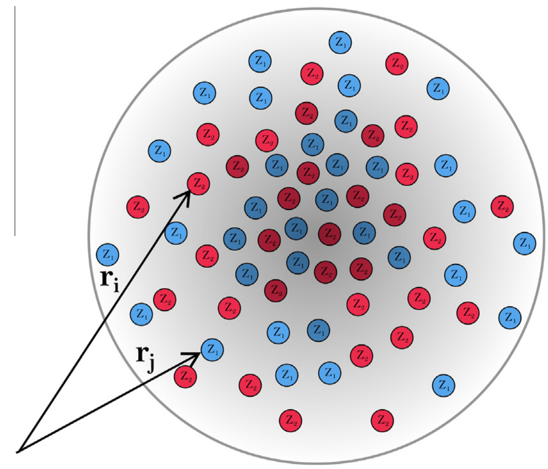

Fig. 1. Bunch consists of two fractures with different charges and different distributions.

Then the distribution of radiated energy per solid angle $d\Omega$ and frequencies is defined by

$$\frac{d^2W(\mathbf{n},\omega)}{d\omega d\Omega}=cr^2|\mathbf{E}^r(\mathbf{r},\omega)|^2=\sum_{\mathbf{g}\neq 0}|\chi_{\mathbf{g}}\mathbf{A}d|^2F_g \tag{7}$$

where

$$F_{\mathbf{g}}(\mathbf{k})=Z_1^2+Z_2^2+2Z_1Z_2\cos((\mathbf{g}-\mathbf{k})(\Delta r_2-\Delta r_1)).$$

For a bunch – mixture consisting of N_1 particles of 1st sort and N_2 particles of the 2nd sort, it is not hard to obtain the expression similar to Eq. (7) with

$$\begin{aligned}F^N_{\mathbf{g}}(\mathbf{k})&=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{p=1}^{N_i}\sum_{l=1}^{N_j}Z_iZ_j\cos((\mathbf{g}-\mathbf{k})(\Delta\mathbf{r}_i^p-\Delta\mathbf{r}_j^l))\\&=\sum_{i=1}^{2}\sum_{p=1}^{N_i}Z_i^2+f^N_{\mathbf{g}}(\mathbf{k})\end{aligned} \tag{8}$$

where

$$\begin{aligned}f_{\mathbf{g}}(\mathbf{k})&=\sum_{i=1}^{2}\sum_{\substack{j=1,\\ j\neq i}}^{2}\sum_{p=1}^{N_i}\sum_{\substack{l=1,\\ l\neq p}}^{N_j}Z_iZ_j\cos((\mathbf{g}-\mathbf{k})(\Delta\mathbf{r}_i^p-\Delta\mathbf{r}_j^l))\\&=Z_1^2N_1N_1\sum_{p=1}^{N_1}\sum_{\substack{l=1,\\ l\neq p}}^{N_1}\cos((\mathbf{g}-\mathbf{k})(\Delta\mathbf{r}_1^p-\Delta\mathbf{r}_1^l))\\&\quad+Z_2^2N_2N_2\sum_{p=1}^{N_2}\sum_{\substack{l=1,\\ l\neq p}}^{N_2}\cos((\mathbf{g}-\mathbf{k})(\Delta\mathbf{r}_2^p-\Delta\mathbf{r}_2^l))\\&\quad+2N_1N_2Z_1Z_2\sum_{p=1}^{N_2}\sum_{\substack{l=1,\\ l\neq p}}^{N_1}\cos((\mathbf{g}-\mathbf{k})(\Delta\mathbf{r}_2^p-\Delta\mathbf{r}_1^l)),\end{aligned} \tag{9}$$

here $\Delta\mathbf{r}_i^p$ describes position of the p-th particle of the i-th sort, and $\Delta\mathbf{r}_j^l$ similarly. The first term in Eq. (9) is responsible for the field of radiation produced by the particles of the first fracture of the bunch, the second term, correspondingly, describes the contribution of the second fracture, and the last term reflects their interference.

Let us put

$$\begin{aligned}&N_1=aN,\\&N_2=(1-a)N,\\&N_i-1\simeq N_i,\end{aligned} \tag{10}$$

distinguishing two different fractures of the charged particles in a beam: first fracture of particles with charge Z_1 in proportion a, and second fracture of particles with charge Z_2 in proportion $(1-a)$. We suppose that the distribution of particles in beam is described by the Gaussian function

$$f_{\mathbf{r}i}(\mathbf{r})=\frac{1}{(\sigma_i\sqrt{2\pi})^3}e^{-\frac{\mathbf{r}^2}{2\sigma_i^2}}, \tag{11}$$

centers of the beam distributions coincide, and σ_i, $i=1,2$ defines the width of distribution for the first and the second fractures, correspondingly. The average value of the first term in sum in Eq. (9) can be written in form

$$\begin{aligned}&\frac{N_1^2Z_1^2}{(\sigma_1\sqrt{2\pi})^6}\int d^3\mathbf{r}_1^p\int d^3\mathbf{r}_1^l\left[e^{-\frac{(\mathbf{r}_1^p)^2}{2\sigma_1^2}}e^{-\frac{(\mathbf{r}_1^l)^2}{2\sigma_1^2}}\cos((\mathbf{g}-\mathbf{k})(\mathbf{r}_1^p-\mathbf{r}_1^l))\right]\\&\quad=N_1^2Z_1^2\exp\left(-\frac{(\mathbf{g}-\mathbf{k})^2\sigma_1^2}{2}\right)\end{aligned}$$

The third term responsible for the field produced by the charges of the different bunches after averaging takes the form:

$$\begin{aligned}&\left\langle 2N_1N_2Z_1Z_2\sum_{p=1}^{N_2}\sum_{\substack{l=1,\\ l\neq p}}^{N_1}\cos((\mathbf{g}-\mathbf{k})(\Delta\mathbf{r}_2^p-\Delta\mathbf{r}_1^l))\right\rangle\\&=\frac{2N_1N_2Z_1Z_2}{(\sigma_1\sqrt{2\pi})^3(\sigma_2\sqrt{2\pi})^3}\int d^3\mathbf{r}_2^p\int d^3\mathbf{r}_1^l e^{-\frac{(\mathbf{r}_2^p)^2}{2\sigma_2^2}}e^{-\frac{(\mathbf{r}_1^l)^2}{2\sigma_1^2}}\cos((\mathbf{g}-\mathbf{k})(\mathbf{r}_2^p-\mathbf{r}_1^l))\\&=2N_1N_2Z_1Z_2\exp\left(-\frac{(\mathbf{g}-\mathbf{k})^2(\sigma_1^2+\sigma_2^2)}{2}\right)\end{aligned}$$

Thus, Eq. (8) takes the form:

$$F^N_{\mathbf{g}}(\mathbf{k})=F_{incoh}+F_{coh} \tag{12}$$

where

$$F_{incoh}=N[aZ_1^2+(1-a)Z_2^2], \tag{13}$$

$$F_{coh}=N^2\left[aZ_1e^{-\frac{(\mathbf{g}-\mathbf{k})^2\sigma_1^2}{2}}+(1-a)Z_2e^{-\frac{(\mathbf{g}-\mathbf{k})^2\sigma_2^2}{2}}\right]^2 \tag{14}$$

Coherent part of radiation becomes comparable to the incoherent part when

$$N\exp\left(\frac{-(\mathbf{g}-\mathbf{k})^2\sigma^2}{2}\right)\approx 1 \tag{15}$$

This can be achieved for very small the beam size or for a comparatively big period of periodical structure. The former is feasible for attosecond bunches [7,8], the latter needs using of artificial periodic structures with a desired period.

It is interesting that the expressions obtained show that even incoherent radiation gives possibility to determine the proportion a describing fracture of the constituents of the whole bunch. Indeed, if F_{coh} is small we can still determine characteristics of the bunch by the following expression with help of Eq. (13):

$$a=\frac{I/N-Z_2^2}{Z_1^2-Z_2^2} \tag{16}$$

where the value

$$I=\left(\frac{dN}{d\Omega}\right)_{bunch}\Bigg/\left(\frac{dN}{d\Omega}\right)_{single} \tag{17}$$

can be measured from the peak of PXR.

Please cite this article in press as: A.Y. Savchenko et al., Parametric X-ray radiation from composite bunches, Nucl. Instr. Meth. B (2015), http://dx.doi.org/10.1016/j.nimb.2015.04.043