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Interference effects in angular and spectral distributions of X-ray Transition Radiation from Relativistic Heavy Ions crossing a radiator: Influence of absorption and slowing-down

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ABSTRACT

Theoretical analysis and representative calculations of angular and spectral distributions of X-ray Transition Radiation (XTR) by Relativistic Heavy Ions (RHI) crossing a radiator are presented taking into account both XTR absorption and RHI slowing-down. The calculations are performed for RHI energies of GSI, FAIR, CERN SPS and LHC and demonstrate the influence of XTR photon absorption as well as RHI slowing-down in a radiator on the appearance/disappearance of interference effects in both angular and spectral distributions of XTR.

1. Introduction

In our previous works [1–7] we have shown that the RHI slowing-down leads to the appearance of very specific diffraction-like structure in both spectral and angular distributions of the Cherenkov radiation (ChR) compared to the standard Tamm–Frank theory. In this paper we consider the influence of RHI slowing-down on another type of radiation from RHI – X-ray Transition Radiation (XTR).

In the standard theory of XTR [8–11] two interfering XTR waves are emitted at both entrance and exit of the radiator by a charged particle moving with a constant velocity. For RHI the influence of slowing-down (ionization energy loss) is not negligible – the RHI velocity decreases during its penetration into a radiator, therefore, two XTR waves are emitted at slightly different velocities when crossing the vacuum–radiator and radiator–vacuum boundaries. In addition, the first XTR wave results to be partially absorbed propagating down to the radiator exit. Finally, due to the velocity decrease, the RHI penetration time into a radiator becomes greater than in the standard XTR theory [8–11], slightly changing the phase shift between two XTR waves. These three factors, namely, partial absorption of the XTR wave in radiator, different velocities at the entrance and exit (defining two formation lengths), and extended penetration time, lead to changing the condition of constructive interference between two XTR waves compared to the standard (without slowing-down) theory of XTR [8–11]. In this paper, new formula for spectral–angular distribution of XTR is suggested, which takes into account the RHI slowing-down in radiator. The representative calculations of both angular and spectral distributions of XTR by RHI crossing a radiator are performed, taking into account both XTR photons absorption and RHI slowing-down in a radiator. The key parameters here are: (a) traditional ones – the plasma frequency of the radiator material, XTR photon energy, XTR photon attenuation length (or linear absorption coefficient), radiator thickness; (b) the new one – the stopping power of RHI in a radiator, which is a complicated function of the RHI energy, its charge and mass.

2. XTR in a radiator: general formulae

If RHI velocity remains constant during its penetration into a radiator, the spectral–angular distribution of XTR radiation intensity (assuming normal RHI incidence condition) into a unit of solid angle $d\Omega$ and frequency interval $d\omega$ can be written in a standard form [8]:

$$\frac{d^2W}{d\hbar\omega d\Omega} = F_0 \cdot F_1. \tag{1}$$

Here, F_0 – describes the generation of the XTR at a single radiator boundary:

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$$F_0(\beta) = \left(\frac{\alpha Z^2\omega^2\sin^2\theta}{16\pi^2c^2}\right)(L_0(\beta) - L_1(\beta))^2. \quad (2)$$

Here, c – is the speed of light in vacuum, Z – the charge of RHI, $\alpha = e^2/\hbar c \cong 1/137$ – the fine structure constant, θ – the XTR emission angle, ω – the frequency of the emitted photon. The quantities L_0, L_1 are the XTR formation lengths in a vacuum and a radiator material, respectively [8–11]:

$$L_i(\beta) = \frac{2\beta c}{\omega(1-\beta\cos\theta\sqrt{\varepsilon_i(\omega)})}, i = 0,\ 1. \quad (3)$$

Here, $\varepsilon_0 = 1$ and $\varepsilon_1(\omega)$ are the dielectric permittivities of vacuum and radiator, respectively. The refractive index $n = n(\omega)$ is connected with dielectric permittivity by the relation: $n_i = n_i(\omega) = \sqrt{\varepsilon_i(\omega)}$. In the X-ray spectral region, $\varepsilon_1(\omega)$ is less than one and may be represented by a simple formula (derived in so-called free electron gas approximation [8–11]):

$$\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \quad (4)$$

Here, $\omega_p^2 = 4\pi\frac{Z}{A}\rho N_A r_0 c^2$ is the plasma frequency of electron gas, Z, A and ρ – are the atomic number, atomic mass number and density of the radiator material, r_0 – is the classical electron radius.

The second function in Eq. (1), F_1 – interference multiplier – arises due to coherent summation of two XTR waves emitted at vacuum–radiator and radiator–vacuum boundaries in a following way [8]:

$$F_1(\beta) = 1 + \exp\left(-\frac{\mu L}{\cos\theta}\right) - 2\exp\left(-\frac{\mu L}{2\cos\theta}\right)\cos\left(\frac{2L}{L_1}(\beta)\right) \quad (5)$$

and μ – is the X-ray linear absorption coefficient of the radiator material, L – is the radiator thickness.

The Eq. (1) may be simply obtained through the summation of the two XTR waves of the same amplitudes $|I|^2 = F_0$ emitted at the two radiator boundaries. For the first wave one has to take into account absorption in a radiator and for the second one – the phase shift that arises due to the RHI penetration into a radiator:

$$\frac{d^2W}{d\hbar\omega d\Omega} = \left|I\exp\left(-\frac{\mu L}{2\cos\theta}\right) - I\cos\left(i\frac{2L}{L_1(\beta)}\right)\right|^2. \quad (6)$$

The simple algebra results in:

$$\frac{d^2W}{d\hbar\omega d\Omega} = |I|^2\left(1+\exp\left(-\frac{\mu L}{\cos\theta}\right) - 2\exp\left(-\frac{\mu L}{2\cos\theta}\right)\cos\left(\frac{2L}{L_1(\beta)}\right)\right) = F_0F_1. \quad (7)$$

3. XTR in a radiator taking account of RHI slowing-down – calculation scheme

The standard formulae (1)–(7) are valid only if RHI velocities before and after crossing a radiator are equal both in direction and magnitude (rectilinear trajectory). In the case of RHI penetrating into a thin radiator, the multiple scattering is negligible, but the ionization energy loss (slowing-down) can influence the phase difference between XTR waves emitted at two subsequent boundaries with different velocities $\mathbf{v}_1$, $\mathbf{v}_2$ and amplitudes $|I_1(\beta_1)|^2 = F_0(\beta_1)$ and $|I_2(\beta_2)|^2 = F_0(\beta_2)$ respectively. The velocity decrease leads to enhancement of the penetration time Δt_S, which now should be calculated using the real RHI velocity values as it penetrates into a radiator: $\Delta t_S = \int_0^L dx/v(x)$. The two effects due to slowing-down are expected: change of constructive interference between two XTR waves (due to a change of the phase shift) and a change of emission amplitude at the second (exit) boundary.

We have modified the Eq. (6) taking into account the change in the phase shift and emission amplitudes of both XTR waves:

$$\frac{d^2W}{d\hbar\omega d\Omega} = \left|I_1(\beta_1)\exp\left(-\frac{\mu L}{2\cos\theta}\right) - I_2(\beta_2)\cos(i\Delta\varphi_S)\right|^2 \quad (8)$$

and obtained the following equation which takes into account the RHI slowing-down in a radiator and replaces the standard formulae (1)–(5) for XTR spectral–angular distribution:

$$\frac{d^2W}{d\hbar\omega d\Omega} = \left(\frac{\alpha Z^2\omega^2\sin^2\theta}{16\pi^2c^2}\right)\left|(L_0(\beta_1) - L_1(\beta_1))\exp\left(-\frac{\mu L}{2\cos\theta}\right) - (L_0(\beta_2) - L_1(\beta_2))\exp(-i\Delta\varphi_S)\right|^2. \quad (9)$$

Here, both $\beta_{1,2} = v_{1,2}/c$ and the phase shift $\Delta\varphi_S$ of the second wave should be calculated taking account of RHI slowing-down. To obtain an equation for $\Delta\varphi_S$, one can divide the radiator into N equal slices of thickness Δl, assuming $\Delta t_i = \Delta l/v_i$ – is the penetration time into each slice with decreasing velocity v_i. As a result, the phase shift of the second XTR wave contains the real penetration time Δt_S of RHI into a radiator.

$$\Delta\varphi_S = \omega\left(\sum_i\frac{\Delta l}{v_i} - \frac{L}{c}n\cos\theta\right) = \omega\left(\sum_i\Delta t_i - \frac{L}{c}n\cos\theta\right). \quad (10)$$

If $N \gg 1$, the summing in Eq. (9) can be replaced by integration, i.e. $\sum_{i=1}^{N}\Delta t_i \to \int_0^L dx/v(x) = \Delta t_S$.

4. Numerical results – XTR spectral–angular distributions

The influence of the photon absorption on the XTR angular distributions in a Be radiator for two different RHI energies is presented in Fig. 1. The X-ray linear absorption coefficient μ in a Be radiator is calculated according to [12–13].

It is obvious that the XTR photons absorption leads to the "washing out" of interference structure of XTR angular distributions and decreasing of the XTR intensity.

Now let us consider the spectral distributions of XTR from RHI. The key formula which takes into account both the RHI slowing-down in a radiator and XTR absorption remains the same, i.e. Eq. (9), but now we keep constant XTR emission angle and change the XTR photon energy. The other important parameters are: RHI energy, radiator material and thickness and exact values of the linear absorption coefficient for each value of XTR photon energy. Our calculations performed taking account of XTR photon absorption showed that only at very high RHI energy there appears remarkable interference structure, see in Fig. 2 where spectral distributions of XTR from Au RHI with initial energy 3000 GeV/u in a W (tungsten) radiator of two thicknesses are shown. The XTR emission angle is fixed and equals $1/\gamma$, with γ being a relativistic factor of RHI.

For the lower RHI energy region – ~1–30 GeV/u (GSI and FAIR) our calculations showed that there are no interference effects in the XTR spectral distributions at different radiator thicknesses.

The role of RHI slowing-down (compared to XTR photon's absorption into a radiator) for chosen RHI energies – 3000 MeV/u, 9000 MeV/u and 3000 GeV/u in Figs. 1 and 2 was negligible, because the slowing-down practically does not change the RHI energy (velocity) in a thin radiator when only the ionization energy loss is taken into account.

The influence of RHI slowing-down on XTR angular and spectral distributions is presented in Fig. 3. In order to get a reasonable