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A novel deflection shape function for rectangular capacitive micromachined ultrasonic transducer diaphragms

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ABSTRACT

A highly accurate analytical deflection shape function that describes the deflection profiles of capacitive micromachined ultrasonic transducers (CMUTs) with rectangular membranes under electrostatic pressure has been formulated. The rectangular diaphragms have a thickness range of 0.6–1.5 μm and a side length range of 100–1000 μm. The new deflection shape function generates deflection profiles that are in excellent agreement with finite element analysis (FEA) results for a wide range of geometry dimensions and loading conditions. The deflection shape function is used to analyze membrane deformations and to calculate the capacitances between the deformed membranes and the fixed back plates. In 50 groups of random tests, compared with FEA results, the calculated capacitance values have a maximum deviation of 1.486% for rectangular membranes. The new analytical deflection function can provide designers with a simple way of gaining insight into the effects of designed parameters for CMUTs and other MEMS-based capacitive type sensors.

1. Introduction

Capacitive micromachined ultrasonic transducers (CMUTs) have been making significant impact in many fields such as medical diagnostic ultrasound, structural health detecting and real-time monitoring of machinery operation [1,2]. CMUT-based sensors are ideal for these purposes due to their high sensitivity, small size, low mass, long lifetime and low power requirement. A CMUT with a rectangular diaphragm is shown in Fig. 1. It consists of a dielectric spacer-supported clamped rectangular diaphragm and a fixed back plate, which are separated by a thin gap. When an external pressure is exerted onto the sensor, the top membrane will deform, leading to a dynamic change in the capacitance between the deformed membrane and the fixed back plate [3].

Highly accurate analytical deflection shape functions that describe the deflection of deformed CMUT membranes will not only provide the insight into the CMUT design methodology but also ascertain the effect of specific geometry parameters [4–11]. For these reasons, deflection shape functions of square and circular membranes have been widely studied by many authors.

Plate theory is often applied to capture the functional form of the deformation curve. The transverse deflection $\omega(x, y)$ of any point (x, y) on a uniformly loaded diaphragm can be obtained by energy minimization method [12]. However, this method is computationally intensive as solutions of numerous simultaneous nonlinear equations are required. Some simple analytical deflection models, the accuracy of which depends mainly on the deformed membrane shape, have been used to predict the deformed curves of square membranes. The deflection model of a rigidly clamped square membrane under a uniform external pressure was first presented as a cosine-like function as [12]

$$\omega(x,\ y) = \omega_0 \cdot \cos\frac{\pi x}{2a} \cdot \cos\frac{\pi y}{2a}, \tag{1}$$

where a is half the side length of the diaphragm and ω_0 is the center deflection. This function describes the general membrane deflection shape, but not accurately. In order to achieved desirable accuracy, authors in [13] expanded the function with two more terms to yield

$$\omega(x,\ y) = \omega_0\left[1 + 0.4 \cdot \frac{x^2 + y^2}{a^2} + 1.16 \cdot \frac{x^2 y^2}{a^4}\right] \cdot \cos\frac{\pi x}{2a} \cdot \cos\frac{\pi y}{2a}. \tag{2}$$

However, authors in [14] pointed out that function (2) fails to catch the deflection profiles of thick membranes, although it agree well with those of thin and large ones. And a new deflection model was introduced in [14] to cover different cases by squaring the cosine terms and adding a new term.

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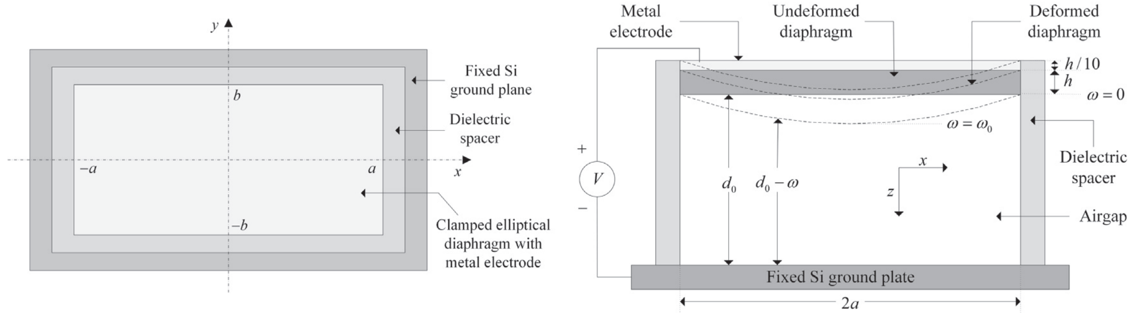

(a) Top view. (b) Cross sectional view [15].

Fig. 1. Top and cross-sectional view of a conceptual CMUT device with rectangular diaphragm.

Authors in [15] further improved the accuracy by developing a new deflection model for square membranes following a two-step process. In the process, the center deflection ω_0 is first obtained by solving a load–deflection model, then the deformed diaphragm shape $\omega'(x, y)$, which is independent from the center deflection, is calculated and multiplied by ω_0 to obtain the complete deflection profile. The load–deflection model of a square diaphragm under a uniform pressure P_M can be expressed as [15]

$$P_M = \left[C_r \frac{\sigma_0 h}{a^2} + C_b \frac{12D}{a^4}\right]\omega_0 + \left[C_s f_s(v) \frac{Eh}{a^4(1-v^2)}\right]\omega_0^3 \tag{3}$$

where σ_0 is residual stress, h is membrane thickness, v is Poisson's ratio. The Poisson's ratio dependent function $f_s(v)$ is given as $f_s(v) = \frac{1-0.271v}{1-v}$. D is the flexural rigidity of the membrane expressed as $D = \frac{Eh^3}{12(1-v^2)^2}$, where E is the Young's modulus of the diaphragm material. The constants C_r, C_b and C_s are determined by comparing the deflection profiles in (3) with finite element analysis (FEA) results [16].

Following the two-step process, the analytical deflection model for CMUT with square membrane was presented as [15]

$$\omega(x, y) = \omega_0 \left(1 - \frac{x^2}{a^2}\right)^2 \left(1 - \frac{y^2}{a^2}\right)^2 \sum_{n=0,1,2}^{N} c_n \left(\frac{x^2+y^2}{a^2}\right)^n. \tag{4}$$

The polynomial basis function was substituted for cosine basis function and higher accuracy was obtained. Function (4) was subsequently used to calculate the capacitance values of CMUTs by formula as

$$C_{Deform} = \varepsilon_0 \iint_A \frac{dxdy}{d_0 - \omega(x, y)}, \tag{5}$$

where d_0 is the gap thickness, and ε_0 is the permittivity of free space, given as $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.

It has been shown that FEA provides highly accurate deflection profiles for CMUT membranes and other MEMS-based transducers [17–21]. But it does not give an insight into the influences of the device geometries on CMUTs as analytical models do. The capacitance calculated by the analytical deflection models (2) and (4) has a good agreement with the FEA results for square diaphragms [15]. However, deflection shape function of rectangular membranes has never been studied, probably due to their complexities in deflection shape. In fact, rectangular membranes are worth studying because they have shown the potential in improving the fill factor and the performance of CMUTs compared with square ones [22,23].

As mentioned above, the accuracy of the analytical deflection model depends not only on the diaphragm's center deflection ω_0, but also on the shape of the deformed membrane $\omega'(x, y)$. This paper focuses on the determination of the deformed diaphragm shapes. We will study a much more general case of membranes and formulate highly accurate deflection shape functions for CMUTs with rigidly clamped rectangular membranes. A data fitting technique is applied to identify the parameters in the deflection shape function by using MATLAB. The effectiveness of the deflection shape function will be illustrated by comparing the predicted deflection profiles with FEA results (using *ANSYS 15.0 software, ANSYS Inc.*). For various clamped CMUT membranes with different geometry dimensions and loading conditions, the new deflection shape function shows excellent agreement with corresponding FEA results.

2. Finite element analysis of CMUT

The 3D FEA model is chosen to simulate the deformation of the membrane. In the finite element simulation, a DC bias voltage (expressed as U) is applied between the electrodes to exert an electrostatic force on the diaphragm. As the bias voltage increases from zero across the membrane and the fixed back plate, the distance between them would decrease until the two plates suddenly snap into contact. This behavior is called the pull-in effect, and the transition voltage is called pull-in voltage. The loading condition on the diaphragm which causes pull-in effect is neglected in the simulation. The device specifications in our simulations are listed in Table 1.

The membrane is modeled by SOLID 186 element while TRANS 126 is employed to apply the electrostatic force on the diaphragm. Considering the symmetrical characteristic of the diaphragm, we use only 1/4 of the diaphragm in the simulation so as to improve the computational efficiency. For boundary conditions, the edges of the membrane are strictly clamped. As shown in Table 2, comparison of center deflections between our simulation results and those in [15] has been conducted to verify the effectiveness of our simulation method. Note that the top electrode is ignored at first to keep consistent with simulations in [15].

It can be observed from Table 2 that our simulation results are nearly the same as those in [15], thus demonstrating that our simplified simulation method insures the accuracy while improving the operational speed.

To simulate the practical operation of CMUTs, a top electrode is added in the finite element model. It has the same width and

Table 1
Device specifications.

Parameter (Unit)	Si_3N_4 diaphragm	Al electrode
Young's modulus (GPa)	169	67.6
Poisson ratio	0.3	0.3555
Density (kg/m^3)	2332	2700
Residual stress (MPa)	50	10