Nuclear Instruments and Methods in Physics Research B 355 (2015) 311–315

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

# Role of spatial dispersion in defining the image of a point charge near the dielectric surface

G.M. Filippov

*Cheboksary Polytechnic Institute, Cheboksary, P. Lumumba-str. 8, Russian Federation*

ARTICLE INFO

*Article history:*
Received 30 November 2014
Received in revised form 11 March 2015
Accepted 12 March 2015
Available online 27 March 2015

*Keywords:*
Polarization field
Mirror image
Space dispersion
Dielectric tube

ABSTRACT

Influence of spatial dispersion on the point charge image near the surface of dielectric or metal is estimated. The polarization field generated by the point charge is calculated. Especially the longitudinal and transverse components of polarization force applied to the point charge moving in the conducting tube are considered. It is shown that the cut-off of the Fourier components of dielectric function in the wave vector space leads to the cut-off in the angular momentum space.

© 2015 Elsevier B.V. All rights reserved.

## 1. Introduction

The channeling phenomena in cylindrically shaped dielectric or metal specimens (capillaries, pores in solid films, nanotubes) are of growing interest for novel developments in science and technology. Channeling of charged particles essentially depends on the polarization capabilities of the metal/dielectric environment. The estimation of polarization forces acting on the channeled particle presumes some physical representations on the inner state of a particle as well as on its trajectory and polarization capabilities of the medium. Within a purely classical treatment of the event, one usually considers the projectile as a point charge, which moves in the vicinity of a solid surface. The charge image sufficiently well describes the polarization field near dielectric or metal surface in the presence of external point charge. Usually, in classical electrodynamics (see, e.g. [1–6]), this problem is solved without taking into account the spatial dispersion of solid polarization properties near the surface. In this paper we investigate the corrections to be done if the medium obey spatial dispersion.

Let calculate the interaction of a point charge with a flat surface of the uniform semi-infinite dielectric medium that based on the concept of the field of surface elementary excitations (field of surface plasmons). In this case, we go beyond the classical electrodynamics using partly quantum mechanical notions. Let the dielectric function of a semi-infinite conducting medium with a plane boundary $z = 0$ to be defined as

$$\varepsilon_\omega = 1 - \Theta(k_c - k)\omega_0^2/\omega^2, \qquad (1)$$

where $\varepsilon_\omega$ is the Fourier transform of the space–time-dependent dielectric function of medium, $k$ is a wave number, $k_c$ is the maximal wave vector of elementary excitations, $\omega$ is the excitation frequency, $\omega_c$ is its maximal value, $\Theta(x)$ – is the Heaviside step function. Due to the spatial dispersion the restriction arises onto the wavelength of the plasma oscillations are to be not less the minimal value $\lambda_c = 2\pi/k_c$. Here the maximal wave vector $k_c$ has the order of the Fermi momentum divided by the Plank constant. The dielectric function (1) qualitatively correct describes the potential of the mirror charge when the point charge is placed near the metal surface. In description of plasmon excitations for a conducting matter it qualitative corresponds to the famous Lindhard approach of the electron gas. However, we do not need any specification of the dielectric function.

The potential of the polarization field can be presented the form

$$\Phi_s^{(pol)}(z \geq 0) = -Z\int_0^{k_c} J_0(k_{||}r)e^{-k_{||}|b+z|}dk_{||}\ . \qquad (2)$$

Here $r = \sqrt{x^2+y^2}$. The point charge $Z$ is assumed to be placed on the distance $b$ in vacuum over the plane boundary on the $z$-axis at $z > 0$. As it follows from (2), at the distances $\sqrt{z^2+r^2} >> k_c^{-1}$ we have

$$\Phi_s(z \geq 0) \approx -Z\int_0^\infty e^{-k_{||}|b+z|}J_0(k_{||}r)dk_{||} = -\frac{Z}{\sqrt{(b+z)^2+r^2}},$$

*E-mail address:* filippov38-gm@yandex.ru

http://dx.doi.org/10.1016/j.nimb.2015.03.036
0168-583X/© 2015 Elsevier B.V. All rights reserved.

and therefore, at large distances the polarization field approximately coincides with the field of the point image charge placed on the other side of the boundary plane and obeying the negative sign. But at small distances $\sqrt{z^2+r^2} \sim k_c^{-1}$ the image charge cannot be considered as a point (in opposite to the external charge). It distributed in the volume with characteristic size $l \sim k_c^{-1}$. In particular, the minimal potential energy of interaction between the external and image charges is not infinite and equals to $U_{\min} = -Zk_c$.

The potential at $z < 0$ is defined with the help of analytic continuation of Eq. (2). Minimum of the image potential is found at the point $z = -b$, $r = 0$. In the neighborhood of the minimum the potential has a more complex behavior than it could be anticipated. In particular, some of its first derivatives in the minimum do not equal to zero. The non-usual behavior of the image potential in the vicinity of the minimum is determined by specific distribution of the charge density in the mirror. First of all, one may conclude that the image potential is symmetric relative to the reflection in the plane $z = -b$. In the region $-b < z < 0$ the potential could be written as

$$\Phi_s(-b<z<0) = -Z\int_0^{k_c} J_0(k_{||}r)e^{-k_{||}(b+z)}dk_{||}\ .$$

This expression being calculated along $z$-axis at $r = 0$ obeys a linear behavior near the origin providing the electric field

$$-\frac{\partial}{\partial z}\Phi_s = (-b<z<0,\ r=0) = \frac{Zk_c^2}{2}\left(1-\frac{2}{3}k_c(b+z)+\cdots\right),$$

is almost constant near the origin. From the physical point of view the linear behavior of the potential on the $z$-axis at the vicinity of the charge image is rather correct and explains the attraction of the external charge to the dielectric/metal surface for any $b > 0$. But in the parallel to the surface direction the electric field suffers the linear behavior

$$-\frac{\partial}{\partial x}\Phi_s(z<0) = (y=0, z=-b, x>\sim 0) = Z\int_0^{k_c} J_1(k_{||}x)k_{||}dk_{||}\Big|_{r=0}$$
$$\approx Z\int_0^{k_c} k_{||}xk_{||}dk_{||} = Z\frac{1}{3}k_c^3x,$$

been equal to zero in the origin.

In this work we consider the motion of the point charge in the tube having the cylindrical symmetry. As a rule, we use the cylindrical coordinate system and the atomic system of units.

## 2. Dielectric tube

Let consider now the case of a dielectric/metal tube (we take in mind that at the nonzero frequency the dielectric formalism is applicable for dielectrics and metals as well). Some important electromagnetic properties of such a specimen were described in the work [5]. In this case (see the cross section in Fig. 1) we assume the external point charge $Z$ is moving with the constant velocity $v$ parallel to the tube's axis at the distance $r_0 < a$ from the axis. We assume also that only in the area 2 the dielectric function is differ from unity, equals to $\varepsilon_\omega$ and takes into account only the time dispersion. In areas 1, 2, 3 we have different solutions for electric displacement potential, which we write as superposition of two sets of linearly independent terms

$$\Phi = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega t}\sum_m e^{im\varphi}\left(\int_0^{\infty}\frac{dk}{2\pi}e^{ikz}R^{+}_{\omega km}(r) + \int_0^{\infty}\frac{dk}{2\pi}e^{-ikz}R^{-}_{\omega km}(r)\right) \tag{3}$$

Here the wave number $k$ assumes to be non-negative. For us it is important to consider the all set of independent solutions to the

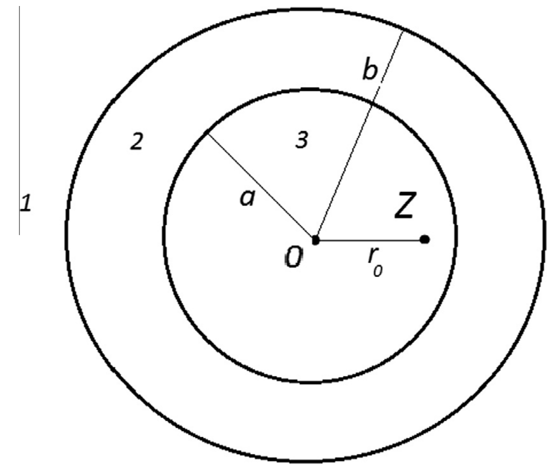


**Fig. 1.** The cross section of the dielectric tube. The point charge $Z$ moves parallel to the tube's axis on the distance $r_0$ from the axis.

wave equation, in particular, for different signs of the longitudinal component of the wave vector $k$. In Eq. (3) this circumstance has been taken into account explicitly. As known, the functions

$R^{\pm}_{\omega km}(r)$ obey the Bessel equation

$$\frac{d^2R^{\pm}_{\omega km}}{dr^2} + \frac{1}{r}\frac{dR^{\pm}_{\omega km}}{dr} - \left(k^2+\frac{m^2}{r^2}\right)R^{\pm}_{\omega km} = -4\pi\rho^{(ext)\pm}_{\omega km}(r). \tag{4}$$

On the boundaries $r = a$, $b$ the continuity of radial derivatives of the potential (3) should be fulfilled, and simultaneously the continuity of quantities $R^{\pm}_{\omega km}/\varepsilon^{\pm}_{\omega}$ should be ensured. We assume the dielectric function has the sign dependence for the wave number.

In the following we use the approach which divides all electromagnetic field in two independent classes, the potential and the vortex. This division is useful in the vacuum electrodynamics (as clearly demonstrated in the book [7]), and could be just more important in the condensed medium. It was used to calculate the stopping power for the projectile moving in dielectric cylinder [8] and is based on two series of the Maxwell equations for the potential and for the vortex fields. We calculate here only contribution of the potential fields to the charge image keeping in mind the application of the theory for comparatively slow ($v \le 1$ a.u.) charged particles. The role of the vortex fields is sufficiently important mostly for the fast projectiles.

In the present work we don't consider the influence of polarization forces on the projectile's trajectory keeping in mind the applications to the sufficiently massive ion (as $Ar^{+q}$, $q > 0$) moving in the tube with approximately constant velocity $\vec{v}$. In this case during the comparatively long time the trajectory could be considered as a straightforward line.

Let assume the point charge is found inside the tube on the distance $r_0(0 < r_0 < a)$ from the axis, at the angle $\phi_0$. In this case $\rho^{(ext)\pm}_{\omega km}(r) = \frac{Z}{r_0}\delta(r-r_0)e^{-im\varphi_0}2\pi\delta(\omega\mp kv)$. Then

$$R^{\pm}_{\omega km}(r) = R^{(ext)\pm}_{\omega km}(r) + \begin{cases} F^{\pm}K_m(kr)+G^{\pm}I_m(kr), & 0\le r\le a \\ B^{\pm}K_m(kr)+C^{\pm}I_m(kr), & a<r\le b \end{cases} \tag{5}$$

Here

$$R^{(ext)\pm}_{\omega km}(r) = -4\pi\int_0^r \left[I_m(kr)K_m(kr') - I_m(kr')K_m(kr)\right]\rho^{(ext)\pm}_{\omega km}(r')r'dr' \tag{6}$$

is the partial solution of inhomogeneous Eq. (4). In consequence of $R^{(ext)\pm}_{\omega km}(0) = 0$ the condition of regularity at the origin $r = 0$ requests to set $F^{\pm} = 0$, therefore,

Download English Version:

https://daneshyari.com/en/article/8040709

Download Persian Version:

https://daneshyari.com/article/8040709

Daneshyari.com