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Quantum resonances in reflection of relativistic electrons and positrons

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ABSTRACT

Calculations based on the use of realistic potential of the system of crystallographic planes confirm earlier results on existence of resonances in reflection of relativistic electrons and positrons by the crystal surface, if the crystallographic planes are parallel to the surface.The physical reason of predicted phenomena, similar to the band structure of transverse energy levels, is connected with the Bloch form of the wave functions of electrons (positrons) near the crystallographic planes, which appears both in the case of planar channeling of relativistic electrons (positrons) and in reflection by a crystal surface. Calculations show that positions of maxima in reflection of relativistic electrons and positrons by crystal surface specifically depend on the angle of incidence with respect to the crystal surface and relativistic factor of electrons/positrons. These maxima form the Darwin tables similar to that in ultra-cold neutron diffraction.

1. Introduction

As is well known, when the angle of incidence θ_0 of fast (relativistic) charged particle with respect to a crystallographic plane is smaller than θ_L – the Lindhard critical angle (see, for example, [2])

$$\theta_L = \sqrt{2|U_0|/E}, \tag{1}$$

the real potential of crystallographic plane can be replaced by continuous (averaged) potential $U(y)$. In formula (1) U_0 is the depth of the continuous potential well (see in Fig. 2), E the particle energy.

Let the crystal surface is parallel to the crystallographic planes, so that the geometry of reflection is as shown in Fig. 1. If the potential of every crystallographic plane is replaced by continuous one, the semi-infinite one-dimensional potential $U(y)$ describes the problem of reflection. Here, the Kronig–Penney-type approximation to real potential $U(y)$ used in Ref. [1] is shown. Within this model, the authors of Ref. [1] showed that the reflection coefficient exhibits resonances as a function of (small) angle of incidence of relativistic particles with respect to the crystal surface and called this mechanism as the band structure mechanism of reflection (quantum effect connected with the band structure of transverse energy levels in the potential $U(y)$). Let us note, that similar problem was considered also in Ref. [3] but for the case of low energy ions.

Since that time up to now, nobody studied these effects experimentally. The goal of our paper is to carry out calculations using more realistic model of $U(y)$ (Section 2) and discuss (propose) possible experiments.

2. The Pöschl–Teller potential for planar channeling of relativistic electrons

The more realistic potential $U(y)$ can be obtained using the Pöschl–Teller potential as $U_1(y)$:

$$U_1(y) = -\frac{U_0}{ch^2 \alpha y}, \tag{2}$$

where U_0 is the depth of the potential well, see in Fig. 2.

To describe planar channeling of relativistic electrons, one needs to solve the Schrödiner equation with relativistic mass γm

$$\hat{H}\phi_n(y) = \left(\frac{\hat{\mathbf{p}}_y^2}{2m\gamma} + U_1(y)\right)\phi_n(y) = \varepsilon_n \phi_n(y), \tag{3}$$

here $\gamma = E/mc^2$ is the relativistic factor, E and ε_n electron longitudinal and transverse energy and $\hat{\mathbf{p}}_y$ an operator of transverse momentum. Fig. 2 shows calculated transverse bound energy levels for relativistic electron with $\gamma = E/mc^2 = 5$ channeled along separate crystallographic plane, the potential of which is described by the single Pöschl–Teller potential well (2) with $U_0 = 25,\ 34$ eV and $\alpha = 2$.

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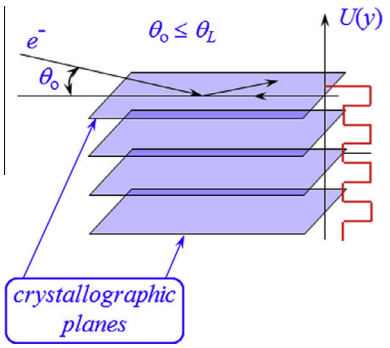

Fig. 1. The geometry of relativistic particle interaction with the crystal surface (semiperiodic Kronig–Penney potential).

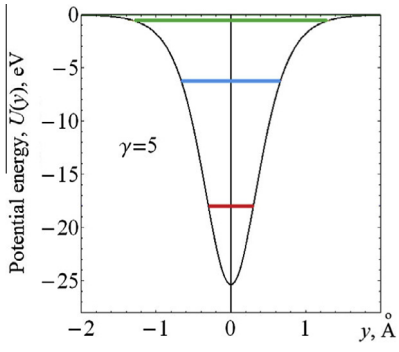

Fig. 2. The Pöschl–Teller potential ($U_0 = 25,\ 34$ eV) for (1 1 0) channeling in Si and planar channeled electron energy levels (relativistic ($\gamma = E/mc^2 = 5$).

With these parameters for $U_1(y)$, summing up over all crystallographic planes, one obtains 1D periodic potential characterized by potential well depth $U_{per} = 21,\ 214$ eV, which is exactly the same as used for exact (numerically calculated) periodic potential of (2 2 0) planes in Si.

For the Pöschl–Teller potential well (2) the formulae for reflection r_0 and transmission t_0 amplitude coefficients are well-known. For our calculations we used the formulae from Ref. [4], making substitution $m \to \gamma m$ to apply them to reflection problem under consideration:

$$r_0 = \frac{\Gamma(s_0 - i\kappa)\Gamma(1 - s_0 - i\kappa)\Gamma(i\kappa)}{\Gamma(s_0)\Gamma(1 - s_0)\Gamma(-i\kappa)},$$
$$t_0 = \frac{\Gamma(s_0 - i\kappa)\Gamma(1 - s_0 - i\kappa)}{\Gamma(1 - s_0)\Gamma(-i\kappa)}, \quad (4)$$

where $\Gamma(z)$ is the gamma function and

$$2s_0 = 1 + \sqrt{1 \pm 8mU_0\gamma/\alpha^2\hbar^2},$$
$$\kappa = \sqrt{2m\varepsilon\gamma/\hbar^2}, \quad (5)$$

here sign plus is for electrons and sign minus for positrons and ε is the electron (positron) transverse energy (continuous energy spectrum).

Fig. 3a and b shows the γ-dependence of reflection coefficient $R = |r_0|^2$ for electrons and positrons at the different angles of incidence θ_0 to a separate crystallographic plane. The drastic difference in R for electrons and positrons is seen. For electrons, the resonances are in principle similar to Ramsauer–Townsend [5] effect.

3. The Pöschl–Teller potential in the problem of reflection of relativistic electrons by a crystal surface

In Ref. [1] the problem of relativistic electrons reflection by a crystal surface was solved in the framework of the Kronig–Penney-type potential. We constructed more realistic new semi-infinite periodic potential $U(y)$ (Fig. 4) which is the sum of separate Pöschl–Teller potentials $U_1(y)$ (Fig. 1). Fig. 4 shows the result of calculation of the electron transverse energy spectrum (bands) using the Schrödiner equation for electrons with relativistic factor $\gamma m = E/mc^2 = 10$.

The electron wave functions in the periodic potential are the Bloch waves and energy spectrum is characterized by the band structure (see, in [3,6–8]). According [8], the allowed energy bands can be determined solving an equation

$$Kd = \mathrm{Arccos}[f(\kappa)],$$
$$f(\kappa) = e^{i\kappa d}(t^2 - r^2 + e^{-2\kappa d})/2t, \quad (6)$$

here K is the Bloch wave vector of electron wave function, d period of the periodic potential (interplanar spacing), r and t are the reflection and transmission amplitude for single well of the periodic potential (single crystallographic plane). In our specific problem these amplitudes depend on relativistic factor and angle of incidence.

Fig. 5 presents the graphic solution of Eq. (6). Since for the real argument $\cos(Kd) \leqslant 1$, the Eq. (6) holds at those values of the electron transverse electron energy ε, at which the function $f(\kappa) \leqslant 1$ ($\kappa = \sqrt{2m\varepsilon\gamma/\hbar^2}$).

According to Refs. [9,10] the reflection amplitude by the periodic semi-infinite Pöschl–Teller potential can be calculated using the following by formula:

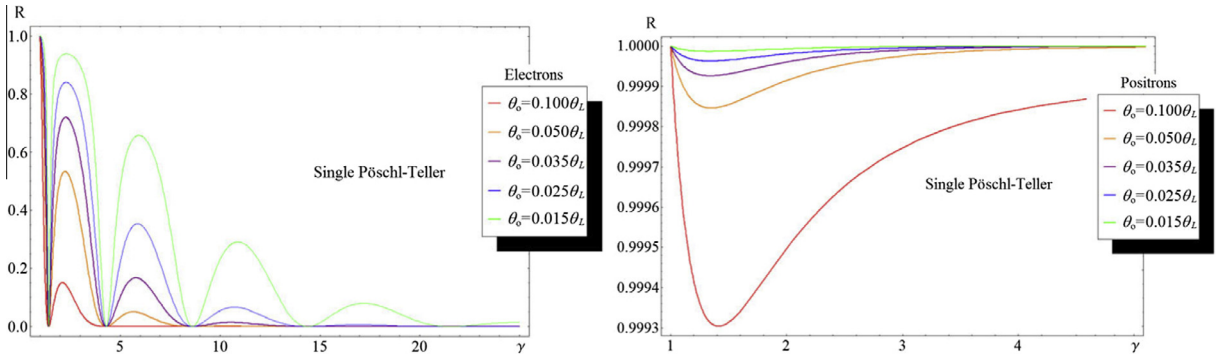

Fig. 3. The Pöschl–Teller potential: γ-dependence of reflection coefficient $R = |r_0|^2$ for the different angles of incidence θ_0 at the crystal surface: (a) for electrons, (b) for positrons.