Nuclear Instruments and Methods in Physics Research B 355 (2015) 351–355

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

About multiple scattering of high energy protons in crystal deflectors

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ARTICLE INFO

Article history:
Received 5 November 2014
Received in revised form 10 February 2015
Accepted 10 February 2015
Available online 26 February 2015

Keywords:
Crystal
Channeling
Multiple scattering

ABSTRACT

The process of multiple scattering of high energy protons in a silicon crystal at its amorphous orientation was studied by simulation of proton trajectories in the model of binary collisions and by a straight simulation of the sequences of proton collisions with atoms when their impact parameters are randomly and uniformly distributed on the symmetry cell for a given crystallography direction. The value of the RMS deflection of multiple scattering obtained by the simulation is in a good agreement with the experiment and more than 15% larger than it follows from the Moliere theory. The obtained RMS deflection used in the Gaussian approach of multiple scattering well describes dechanneling of protons in the frame of the planar potential model. Different number of proton collisions with atoms occurs along the same crystal length for different crystal orientations. However, the change of the collision number is compensated by the corresponding change of the mean square deflection in a single collision. Therefore, multiple scattering is the same for different crystal orientations. The generator of multiple scattering for amorphous crystal orientations was proposed.

1. Introduction

High energy charged particles passing through a matter experience multiple deflections in collisions with atoms. The deflections occur mainly due to scattering by the electric field of atomic nuclei screened by electrons. The theoretical description of multiple scattering is based on the solution of the transport equation considering the particle balance for a given deflection. The multiple scattering theory of Moliere whose description may be found in [1] is the most popular one. The angular distribution of particles due to multiple scattering is close to a Gaussian one because the deflections are formed by the sequences of large number of small deflections. However, the distribution differs from the Gaussian one for large deflection angles because of a contribution of close collisions with atoms to the sequences.

The Gaussian distribution approach is widely used in the experimental practice for the description of multiple scattering of charged particles in the targets. One of the first estimations for the mean square of the multiple scattering angle by atomic nuclei was suggested by Rossi [2]:

$$\bar{\theta}_R^2 = \left(\frac{E_s}{pv}\right)^2 \frac{L}{L_R}, \tag{1}$$

where p, v are the particle momentum and velocity, L is the target length, L_R is the radiation length (for interactions with nuclei without a contribution of electrons), $E_s = 21$ MeV. The estimation of the RMS deflection obtained by fitting the Moliere distribution shows its nonlinearity with the target length L [3]:

$$\theta_o = \frac{13.6\ \text{MeV}}{pv}\sqrt{L/L_R}(1 + 0.038 \ln\,(L/L_R)). \tag{2}$$

Successive collisions with atoms are correlated for the particle passage through a crystal in the channeling conditions. The trajectories of channeled particles are described in the first approximation by the potential averaged along some crystallographic planes or axes [4]. The real crystal potential differs from the averaged one due to the lattice discreteness and thermal vibrations of atoms. This causes deflections of channeled particles, which are considered as a result of their multiple scattering by atomic nuclei. In this case, the intensity of multiple scattering depends on the transverse particle position in a channel. The mean square deflection (MSD) per unit length is described through the local density of atomic nuclei in a channel $P(x)$ formed by thermal vibrations of atoms around the lattice points [5]:

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http://dx.doi.org/10.1016/j.nimb.2015.02.036
0168-583X/

$$\frac{\Delta\bar{\theta}^2}{\Delta z}(x) = P(x)\left(\frac{\Delta\bar{\theta}^2}{\Delta z}\right)_R, \tag{3}$$

where $(\Delta\bar{\theta}^2/\Delta z)_R$ is the MSD value for the amorphous state of the target. Its estimation can be made with the expression (1). The simulation of particle trajectories in the averaged potential of bent crystallographic axes or planes with the step-by-step calculation of multiple scattering in the Gaussian distribution approach with the mean square deflection (3) has been proposed in [6] for the description of channeling effects in bent crystals [7]. The model of the lattice of bent atomic strings or planes gives a good agreement with the results of the experiments on the study of channeling and volume reflection in bent crystals performed at the external beams of the CERN SPS and in the experiments on the SPS beam collimation assisted with a bent crystal. In the last years, simulation codes for these purposes were also developed by the other authors [8–11]. The accurate presentation of large deflections in multiple scattering of high energy protons in a bent crystal for its amorphous orientations, which are not described in the Gaussian distribution approach, is necessary for the studies of the crystal assisted collimation at the SPS and the LHC. Some approximation for the tail of the multiple scattering distribution was suggested in [12]. However, the approximation may describe the real distributions only for some narrow range of the target thickness.

In this paper multiple scattering of 400 GeV/c protons in a silicon crystal for its amorphous orientations was studied by simulation of particle trajectories using the binary collision model [13]. The simulation results were compared with the experiment and with the distribution calculated according to the Moliere theory. The obtained value of the RMS deflection was used then in the planar potential model for the calculation of multiple scattering of protons in channeling conditions according to (3) to study the proton beam deflection by a bent silicon crystal in the experiment [14].

Besides, a straight simulation of multiple scattering of protons in a crystal was performed by the calculation of the sequences of deflections in random collisions of protons with Si atoms when their impact parameters are uniformly distributed on the symmetry cell for a given crystallographic direction. The number of collisions with atoms is different for different crystallographic directions at the same crystal length. The effect of this circumstance on multiple scattering was considered.

2. Multiple scattering of protons in a Si crystal in binary collision model

It is considered that multiple scattering of charged particles in a crystal target for its orientations far from the main crystallographic axes and planes is the same as in the target with amorphous state of the same substance (these crystal orientations are called "amorphous" ones). Therefore, it is assumed that the impact parameter distribution of particles in collisions with the crystal atoms is uniform in this case.

The model of binary collisions [13] was used here to study multiple scattering of high energy protons in a silicon crystal. This model allowed us in [15] to prove possibility of the deflection of high energy charged particles in channeling states by a bent crystal before the real experiment [16]. In this model, the particle trajectories formed in collisions with atoms in a crystal with taking into account the crystal lattice geometry and thermal vibrations of atoms around the lattice points were calculated in detail.

Simulation was made for the passage of 400 GeV/c protons through a 1.94 mm long silicon crystal along its ⟨1 1 1⟩ crystallographic axis (for the conditions of the experiment [14]). Fig. 1a shows the symmetry cells for the ⟨1 1 1⟩ direction in a Si crystal. The central points of the cells correspond to the atomic string

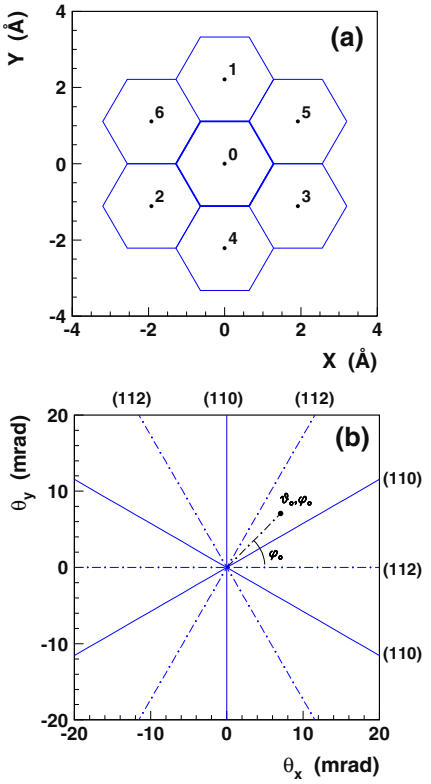

Fig. 1. (a) Schematic presentation of the symmetry cells in a Si crystal along its ⟨1 1 1⟩ axis. The lattice points in the cell centers are also shown. (b) Angular space around the ⟨1 1 1⟩ Si axis. Lines show the directions of the main crystallographic planes passing through the axis. Amorphous orientation used in simulation is shown by the point with angular coordinates (θ_o, φ_o) far from the axis and plane directions.

positions along the ⟨1 1 1⟩ direction. The distance between atoms in the ⟨1 1 1⟩ strings alternates, $d_{a1} = \sqrt{3}d/4$ and $d_{a2} = 3d_{a1}$, where $d = 5.431$ Å is the lattice constant. The atoms of the strings (1–3) and (4–6) are shifted relative to the atoms of the string 0 by $d_{s1} = \sqrt{3}d/3$ and $d_{s2} = 2d_{s1}$, respectively. The deflection angle in the collision with a silicon atom with the impact parameter r was calculated as in [15] in the momentum approximation using the Moliere approach for the atomic potential [17]:

$$\theta(r) = \frac{2Ze^2}{pva}\sum_{i=1}^{3}\alpha_i\beta_i K_1(\beta_i r/a), \tag{4}$$

where a is the screening length, Z is a silicon nucleus charge, K_1 is the modified Hankel function of the first order, α_i, β_i are the Moliere potential coefficients. The table of the values $r\theta(r)$ were calculated according to (4). The proton deflection angle for a given impact parameter r was found from the table by interpolation as in [18] (the inaccuracy in this case is smaller than 1% already with the table step of 10^{-3} Å).

Fig. 1b shows the angular space around the ⟨1 1 1⟩ crystallographic axis of a silicon crystal. The lines show the directions of the main crystal planes passing through the axis. The direction of the incident beam should be far from the axis and plane directions to avoid the potential scattering of protons. So, the contribution of the potential scattering by a crystal plane decreases with the orientation angle θ_{xo} as:

$$\Delta\theta_p(\theta_{xo}) = \theta_{xo}\left[1 - (1 - \theta_c^2/\theta_{xo}^2)^{1/2}\right], \tag{5}$$

where $\Delta\theta_p$ determines the difference of the exit angles for particles with the same transverse energy which exit the crystal at two