On the duration of shock loading and yield strength

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Thermal atomic motion in a crystal is interpreted as a certain rapidly oscillating stress field called a fluctuation field. A fluctuation field theory is developed in the context of solid state physics and statistical physics. The theory is applied to the description of dislocation motion in a crystal at external stress lower than the threshold required for dislocation motion. The dislocation motion is thus due to the joint action of external and fluctuation stresses. The shock pulse duration at which stress fluctuations have no chance (with 0.99 probability) to reach the level required for dislocation motion is calculated. With this pulse duration, the material does not experience plastic deformation, whereas with a longer loading pulse at the same stress it does. The effect, i.e., the absence of plastic deformation with a short loading pulse, can be eliminated by increasing the stress in the pulse. This suggests that the material yield strength increases with decreasing the duration of shock loading.

Keywords: dislocation, stress fluctuations, yield strength, shock loading

1. Introduction

The basic mechanism of plastic deformation of crystalline materials is dislocation motion. The yield strength of crystalline materials, as a rule, displays a descending temperature dependence and hence it is thermal atomic oscillations that are conducive to dislocation motion. Thus, dislocation motion is due to the joint action of applied stress and thermal atomic oscillations. The effect of thermal atomic oscillations on each individual dislocation segment is discrete in time: the time intervals on which they assist and fail to assist the dislocation motion are alternating. According to the laws of statistical physics, the above time intervals alternate with a very high rate; therefore their time-discreteness escapes detection in ordinary experiments on plastic deformation of materials. However, intensive recent research in the mechanical properties of materials under supershort shock loading [1] brings up the reasonable question: How short must the duration of external loading be that thermal atomic oscillations have no time, to a high probability, to initiate dislocation motion?

In this work, we calculate the loading pulse duration at which thermal atomic oscillations required for dislocation motion have no chance to occur (to a 0.99 probability) on the basis of the Einstein–Debye fluctuation theory and data of [2]. With this loading pulse duration, the material does not experience plastic deformation, whereas with a longer pulse at the same stress it does. The effect, i.e., the absence of plastic deformation with a short loading pulse, can be eliminated by increasing the stress in the pulse and this suggests that the material yield strength increases with decreasing the duration of shock loading.

2. Stress field due to thermal atomic oscillations

Let us consider a crystal as an elastic homogeneous isotropic solid that contains dislocations of one slip system. An immobile dislocation is set in motion providing that the tangential stress tensor component operating in its vicinity and along slip plane in the Burgers vector direction is greater in the absolute value than a certain threshold σ_0 (representative of the resistance to dislocation motion). Let us analyze the case where the above external stress component σ_{ext} falls short of the threshold and is positive. Thus, $0 < \sigma_{ext} < \sigma_0$, and the aid of thermal atomic oscillations is required to actuate the dislocations.

Let us treat thermal atomic oscillations as a certain rapidly oscillating stress field — call it a fluctuation field — universally present in the solid and dependent on its temperature.

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The fluctuation stress field, like an ordinary stress field, is bound to obey the system of equations of elasticity. However, our further discussion makes clear that the dislocation motion is governed by only quite definite values of this field in small solid subregions. Hence most of the fluctuation stress field has no effect on dislocation motion and its detailed form is of no importance for the description of the process. In this context, formulation and solution of the elasticity problem for the fluctuation stress field is omitted in the discussion and we consider this field as a set of individual stress fluctuations (or "flashes" of stress fluctuations) arising in certain solid subregions. These terms are used to emphasize the time and space discreteness of the field. The field parameters are specified with resort to the known concepts of solid state physics and statistical physics.

For the fluctuation field, we set the simplifying assumptions similar to those taken in [2]:

– the fluctuations of all six independent stress tensor components are independent of each other and are alike in probability distribution;

– the solid subregions involved in fluctuations are spherical; the subregion diameters *D*, unlike those taken in [2], are different and can assume any value from the lattice cell parameter *a* to the minimum linear dimension of the solid *L*[1];

– the fluctuation stress field is homogeneously inside each subregion involved in fluctuation;

– the duration τ_f of a fluctuation "flash" in a subregion of diameter *D* is equal to the time it takes for an elastic wave to travel a distance *D* with a velocity of sound c_t:

$$\tau_\text{f} = \frac{D}{c_\text{t}}. \tag{1}$$

By analogy with the known Einstein and Debye heat capacity models [3], we consider that the fluctuation stress field in the solid is produced by independent oscillators each of which has a certain frequency ν_f, different for different oscillators and the number of which is equal to the number of internal degrees of freedom of the solid $3N-6 \approx \approx 3N$ (*N* is the number of atoms in the solid). The frequency ν_f is taken to be the reciprocal of the fluctuation duration τ_f:

$$\nu_\text{f} = \frac{1}{\tau_\text{f}} = \frac{c_\text{t}}{D}. \tag{2}$$

The quantity ν_f is termed the oscillator frequency or the fluctuation frequency. From the above properties and from formula (2) it follows that to each oscillator corresponds a certain diameter *D* of subregions in which it generates a fluctuation stress, and to different oscillators correspond different diameters.

[1] The minimum linear dimension of a solid in the case where the latter is a rectangular parallelepiped is the minimum length among the lengths of its three edges issuing from one vertex; if the solid is a cylinder, it is the minimum quantity among its height and diameter; if the solid is a plate, it is its thickness.

Of concern to us is the tangential fluctuation stress tensor component affecting the dislocation motion. We have $3N$ oscillators and equally distributed fluctuations of six independent stress tensor components; hence, the tangential fluctuation stress tensor component of interest is generated by $N/2$ oscillators. It is assumed that half of the oscillators produce stress of one sign, and the other produce stress of opposite sign. We consider only $N/4$ oscillators responsible for the tangential stress component codirectional with the tangential external stress component σ_ext.

Note that the fluctuation stress is capable of moving dislocations not only in the direction of external stress, but in the opposite direction as well. However in the latter case, the fluctuation stress is bound to be greater (in the absolute value) than that in the former case due to the necessity to overcome the external stress along with the resistance to dislocation motion. Moreover, the probability of fluctuation decreases rapidly with increasing its value, and hence with a rather high external stress, the probability of fluctuations moving the dislocations in the direction of external stress is so much higher than the probability of fluctuations moving them in the opposite direction that the latter can be ignored.

Let *Q*(*D*) be the diameter distribution density for solid subregions involved in fluctuations. Because the diameter range and the oscillator range have one-to-one correspondence, *Q*(*D*) is simultaneously the oscillator distribution density. By analogy with the Debye model [3], let the number of oscillators with frequencies in the range $[\nu_\text{f}, \nu_\text{f} + \text{d}\nu_\text{f}]$ be proportional to $\nu_\text{f}^2 \text{d}\nu_\text{f}$ (in the linear approximation in $\text{d}\nu_\text{f}$). Then in view of formula (2), we have

$$Q(D)\,\text{d}D = \text{const}\,(c_\text{t}^3 / D^4)\,\text{d}D.$$

Once the constant in this equality is found from the condition that the number of oscillators is $N/4$ and $D \in [a, L]$ (where *a* is the lattice cell parameter and *L* is the minimum linear dimension of the solid), we conclude the following.

The number of oscillators initiating fluctuations of the tangential stress component in subregions of diameter $[D, D+\text{d}D]$, is

$$Q(D)\,\text{d}D = \frac{3Na^3L^3}{4(L^3-a^3)D^4}\text{d}D \approx \frac{3Na^3}{4D^4}\text{d}D \tag{3}$$

(in the linear approximation in d*D*). In the above equation, $a \ll L$.

Let the tangential stress tensor component in question reaches σ_f in a certain subregion of volume Ω (according to the assumption taken, it is homogeneous in this subregion, $\sigma_\text{f} > 0$). Hence, the corresponding elastic strain tensor component $\varepsilon_\text{f}^\text{e} = \sigma_\text{f}/(2G)$ (*G* is the shear modulus). The elastic energy of this stress fluctuation is

$$U_\text{f} = 2\int_{(\Omega)}\text{d}\Omega \int_0^{\sigma_\text{f}} \sigma\,\text{d}\varepsilon^\text{e} = \frac{\sigma_\text{f}^2\Omega}{2G}, \tag{4}$$

where the subregion volume and the subregion itself are