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Determining evolutionary spectra from non-stationary autocorrelation functions

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ABSTRACT

For non-stationary stochastic processes, the classic integral expression for computing the autocorrelation function from the evolutionary power spectral density (evolutionary spectrum) developed by Priestley is not invertible in a unique way. Thus, the evolutionary spectrum cannot be determined analytically from a given autocorrelation function. However, the benefits of an efficient inversion from autocorrelation to evolutionary spectrum are vast. In particular, it is more straightforward to estimate the autocorrelation function from measured data, yet efficient simulation depends on knowing the evolutionary spectrum. This work examines the existence and uniqueness of such an inversion from the autocorrelation to the evolutionary spectrum under a certain set of conditions. It is established that uniqueness of the inversion is likely although it is not proven. A methodology is presented to determine the evolutionary spectrum from a prescribed or measured non-stationary autocorrelation function by posing the inversion as a discrete optimization problem. This method demonstrates the ability to perform the inversion but is computationally very expensive. An improved method is then proposed to enhance the computational efficiency and is compared with some established optimization methods. Numerical examples are provided throughout to demonstrate the capabilities of the proposed methodologies.

1. Introduction & motivation

Through increases in both the computational power and efficiency of algorithms available to scientists and engineers, it is becoming increasingly possible to consider problems in a fully stochastic framework that were once only approachable deterministically. Monte Carlo simulation remains at the forefront of the analysis methods for its robustness. In fact, for many classes of problems (and several of practical interest), Monte Carlo simulation is the only option to fully characterize the system. This may be true, for example, in evaluating the structural response to seismic ground motion; characterizing the morphology of statistically non-homogeneous random media – like Functionally Graded Materials (FGMs), evaluating the response of bridges to wind loads; and is particularly true when strong nonlinearities are present.

The bedrock of Monte Carlo methods is the efficient and accurate simulation of sample realizations. For the examples previously listed, this translates to generating sample earthquake time histories, material morphologies, and wind velocity time histories/fields, respectively. The Spectral Representation Method (SRM) [1] is widely used for the simulation of sample realizations of Gaussian stochastic processes and recent developments have widely enhanced its applicability for non-Gaussian processes [2–4] by coupling with Grigoriu's translation process theory [5]. SRM is the method employed in this work. However, there are many other simulation techniques available (e.g. Autoregressive Moving Average (ARMA) models [6–11] and the Karhunen–Loève expansion [12–15], among others) that will not be discussed in detail in this work. The SRM, in particular, is commonly used in the fields of civil engineering and applied mechanics, due partly to its nice physical interpretation afforded by the trigonometric basis functions that give the power spectral density function units of power at different frequencies. Additionally, it is comparatively mathematically simple and numerically easy to implement, as the simulated samples are generated by a finite sum of cosine functions or fast Fourier transform.

This work is specifically concerned with non-stationary stochastic processes.[1] A process is said to be non-stationary if its statistical properties vary in time. There are, of course, varying degrees of non-stationarity. We specifically focus on a class of non-stationary processes whose second-order statistics (correlations/

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[1] The terms "non-stationary" and "non-homogeneous" are used interchangeably throughout this paper, as the theory utilized herein is analogous for both random processes and random fields.

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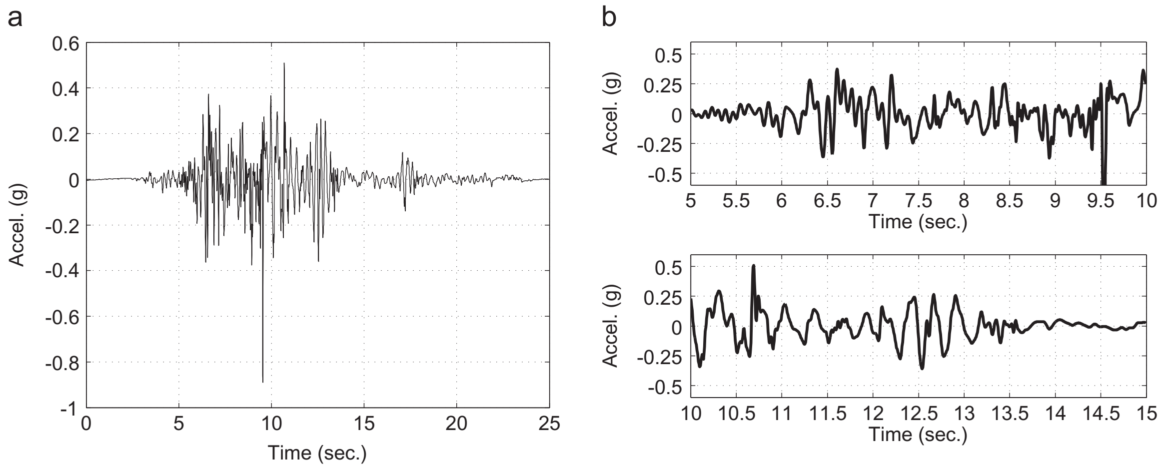

Fig. 1. Example seismic ground acceleration: 1989 Loma Prieta earthquake. Data source: Pacific Earthquake Engineering Research Center (PEER) [16], Record #: NGA0779, ATH: LOMAP/LGP-UP. (a) Ground acceleration and (b) zoom details on ground acceleration.

power spectra) are time dependent. In the physical interpretation of the SRM, this means that the frequency content is time varying. Practical examples of non-stationary processes are ubiquitous, even when limited just to the fields of civil engineering or applied mechanics. Seismic ground motion time histories, for example, exhibit both amplitude and frequency modulation; as demonstrated in Fig. 1 which shows a clear amplitude and frequency modulation. The latter is more readily observed in Fig. 1(b) where it can be seen, qualitatively, that the predominant frequencies in the two displayed time windows are significantly different – the top having higher frequency content than the bottom. As another example, Functionally Graded Materials (FGMs) are a class of composites where a gradient exists in stochastic material morphology and properties. FGMs occur both naturally (e.g. bamboo possesses a radially increasing fiber density to resist wind induced bending moments [17,18]) and are synthetically engineered (Fig. 2 shows a two phase aluminum – high density polyethylene composite with varying volume fraction). To capture this spatial variation, the morphology may be characterized by a non-homogeneous stochastic field.

Modeling non-stationary processes represents a particularly challenging problem for which several approaches have been proposed. Limiting these approaches to those possessing a direct physical interpretation (i.e. frequency-based), we see that several different theories for modeling the evolving spectral characteristics exist including, most notably the Instantaneous Power Spectrum [19], the Wigner–Ville Spectrum [20–22], wavelet transforms [23–30], and Priestley's theory of evolutionary power [31]. Among these, Priestley's theory is particularly useful because its defining quantity, the evolutionary power spectral density (or evolutionary spectrum – ES) preserves the features of the classical stationary power spectral density as discussed in the following.

Consider a zero mean non-stationary stochastic process $X(t)$ possessing autocorrelation function (ACF) $R(s,t)$ that admits a representation of the form [31]:

$$R(s,t) = E[X(s)X(t)] = \int_{-\infty}^{\infty} \phi(s,\omega)\phi^*(t,\omega)\, d\mu(\omega) \tag{1}$$

where $\phi(t,\omega)$ represents a “family” of functions defined on the real line and $\mu(\omega)$ is a measure also defined on the real line. In general, $X(t)$ can be expressed as

$$X(t) = \int_{-\infty}^{\infty} \phi(t,\omega)\, dZ(\omega) \tag{2}$$

where $Z(\omega)$ is an orthogonal process with

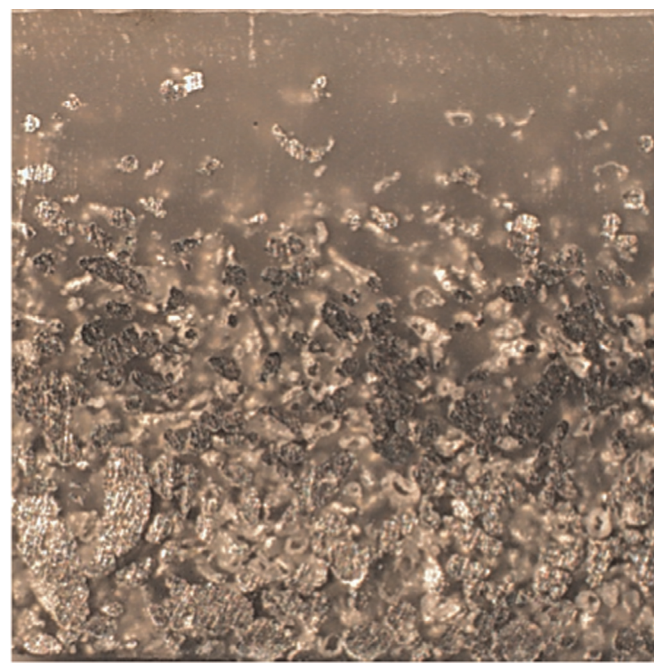

Fig. 2. Sample functionally graded material: aluminum particulates in a high density polyethylene matrix (AL-HDPE). Image courtesy of Po-Hua Lee, Columbia University, 2013.

$$E[|dZ(\omega)|^2] = d\mu(\omega) \tag{3}$$

For processes defined on a finite interval (i.e. $0 \le t \le T$), such a representation always exists with $\phi(t,\omega)$ denoting the eigenfunctions of the covariance kernel; which serves as the basis of the Karhunen–Loève decomposition. The theory of evolutionary power, on the other hand, enforces a family of amplitude modulated complex exponentials such that

$$\phi(t,\omega) = A(t,\omega)e^{i\theta(\omega)t} \tag{4}$$

where $A(t,\omega)$ is the so-called modulating function that can be expressed as

$$A(t,\omega) = \int_{-\infty}^{\infty} e^{i\theta t}\, dH(\omega,\theta). \tag{5}$$

If $|dH(\omega,\theta)|$ possesses an absolute maximum at $\theta=0$, the process is called oscillatory. Thus, if $\theta(\omega)$ is a single-valued function of ω, the