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Filter models for prediction of stochastic ship roll response

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ABSTRACT

In this paper, the shaping filter technique is introduced to study the stochastic roll response of a vessel in random beam seas. Specifically, the roll motion is described as a single-degree-of-freedom (SDOF) model in which the stationary random wave excitation term is approximated as the output of a shaping filter, i.e. a filtered white noise process. Therefore, the original SDOF model can be extended into a four-dimensional (4D) and a six-dimensional (6D) dynamic system, respectively, when this second-order nonlinear differential equation couples with the second-order linear filter and the fourth-order linear filter used to model the random wave excitation. Basically, the fourth-order linear filter provides a better approximation, but the subsequent dynamic system would be more complicated to tackle. In this regard, the 4D coupled system can be viewed as a Markov system whose probability properties are governed by the Fokker–Planck (FP) equation. Furthermore, the 4D path integration (PI) method, an efficient approximate technique based on the Markov property of the dynamic system, is applied to solve the 4D FP equation and then the response statistics are derived. In contrast, the statistics of the roll response in the 6D extended system are evaluated directly by a 6D Monte Carlo simulation. The advantages of the 4D PI method as well as the feasibility of simplifying the 6D system by the 4D system in terms of determining the statistics of high-level roll response are demonstrated through various cases corresponding to different sea states. Furthermore, the influence of external excitations on the statistics of high roll response levels is also illustrated by these cases.

1. Introduction

Large amplitude roll motion in realistic seas is a serious threat to ship stability because it can lead to damage or even capsizing of the vessel. However, assessing the response statistics of large amplitude roll motion excited by random wave excitations is a challenging task. In this situation, the dynamic behaviors of the roll motion, especially the high-level roll responses, are influenced significantly by various effects which are nonlinear in nature or inherently random. Principally, elaborate theoretical models as well as appropriate mathematical techniques are essential to analyze this kind of problem [1].

For the cases of beam seas, the roll motion can be treated as decoupled from other motion modes and governed by a single-degree-of-freedom (SDOF) model in which the nonlinear effects associated with damping and restoring terms as well as the random wave excitation term are all incorporated [2]. The methodology based on the theory of Markov diffusion processes to evaluate the stochastic roll response is attractive since the probability distribution of the roll response is governed by the Fokker–Planck (FP) equation. However, the theory of Markov processes is only valid for the dynamic systems excited by Gaussian white noise. On the other hand, for the SDOF model of roll motion, the external excitation is generally non-white.

Nevertheless, the random wave excitation term is a stationary process with appropriate spectral density and it can be modeled as filtered white noise or colored white noise via the shaping filter technique. Spanos [3–5] was the pioneer in introducing the filter algorithms to approximate the wave elevation and wave kinematics. Subsequently, the filter approaches were widely used to model the wave loads and evaluate the responses of the nonlinear systems in the field of ocean engineering (e.g. [6–11]). In this paper, two different filter models, a second-order linear filter and a fourth-order linear filter, are used to approximate the random wave excitation process. Correspondingly, a four-dimensional (4D) and a six-dimensional (6D) extended dynamic system are created when the original SDOF model, a second-order nonlinear differential equation, is coupled with the filters used to generate filtered white noise. In this regard, the coupled dynamic systems can be viewed as excited by Gaussian white noise processes.

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However, analytical solutions of the FP equation are known only for some linear systems and a very restricted class of nonlinear systems. Direct numerical methods aiming to solve the low-dimensional FP equations, such as the finite-element method [12] and the finite difference method [13] are hardly feasible for the nonlinear extended dynamic systems with high-dimensional FP equations. In these cases, the so-called "curse of dimension" comes into play which means that difficulties arise due to the processing capacity as well as the storage needed for the computation increases dramatically with the dimension of the FP equations. Therefore, several alternative techniques are developed to provide approximate solutions to the FP equations, such as the stochastic averaging method [2], the local statistical linearization method [9], the Gaussian closure method [7,10], etc.

The path integration (PI) method is an efficient approximation to find the stationary and non-stationary response probability density functions (PDF) of the Markov dynamic systems. This method is based on the Markov property of the dynamic system and the evolution of the response PDF is computed in short time steps via a step-by-step solution technique. Specifically, according to the Chapman–Kolmogorov equation, the response PDF at a given time instant can be obtained when the response PDF at an earlier close time instant as well as the conditional PDF with a Gaussian form are already known. The main advantage of the PI method and the Markov dynamic system is that a host of accurate and useful response statistics can be obtained within one calculation. Wehner and Wolfer [14] were the first to apply the numerical PI method to solve nonlinear FP equations and then various PI procedures were developed and applied to address certain problems in the area of engineering, e.g., [15–20] etc. In the field of marine engineering, Naess and Johnsen [21] developed a three-dimensional (3D) PI procedure to estimate the response statistics of moored offshore structures and this research topic as well as the PI technique were extended in Karlsen's research work [22]. In their work, the PI method has been shown to be capable of providing satisfactory estimation of the stochastic response, even in the tail region for low probability levels. Recently, this algorithm was successfully extended to 4D for studying the stochastic roll response of a ship in random beam seas [23,24].

Unfortunately, the PI method does suffer from a curse of dimensionality problem. Currently, some 4D problems can be analyzed by the PI method at an acceptable computational cost. However, the associated computation for the 6D coupled nonlinear system is unaffordable at present. On the other hand, it may be noted that the Monte Carlo simulation method does not suffer critically from the curse of dimensionality problem since the statistics of the response are obtained directly from the realizations. Basically, the Monte Carlo simulation is the simplest and most versatile way to determine the response statistics of the dynamic systems. For the cases of ship rolling, the nonlinear damping and restoring moments as well as the time-dependent wave excitation term can be directly dealt with. Even though Monte Carlo simulation enables the empirical estimation of response statistics for the 6D augmented system to be determined, this work, on the basis of straightforward counting, is only a brute force alternative and has its drawback at the same time. It cannot provide information at the same level of detail as the PI method [25]. Particularly, when the Monte Carlo method is applied to estimate the statistics of large roll response with low probability levels or evaluate different kinds of response statistics in one step, the associated computational cost as well as the efficiency would be sacrificed in practice.

In this paper, the upcrossing rate for high-level roll responses and the probabilities of exceedance are of central importance in evaluation of the response statistics. The feasibility of replacing the 6D extended system by the 4D coupled system in terms of determining the above two aspects will be discussed. The main scheme of the proposed work is shown in Fig. 1. Specifically, for different sea states, the response statistics calculated by the 4D PI method will be compared with that evaluated by the pertinent 6D Monte Carlo simulation. If the simplicity is practicable, the 4D dynamic system and the efficient 4D PI method, with its advantage in providing different kinds of response statistics within one calculation, can be applied to address the challenge of determining the stochastic roll response subjected to random wave excitation. The numerical approaches as well as the results and conclusions obtained in this work hopefully can provide useful references for ship stability research and stochastic dynamic analysis of nonlinear systems.

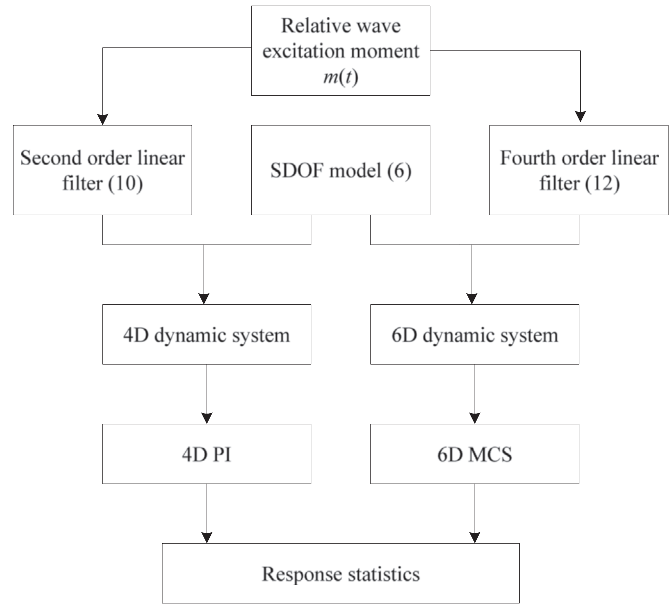

Fig. 1. Main scheme of the work.

2. Mathematical model of roll motion

By neglecting coupling, the rolling behavior of the vessel in random beam seas can be represented by the following SDOF equation:

$$I\ddot{\theta}(t) + B(\dot{\theta}(t)) + C(\theta(t)) = M(t) \tag{1}$$

where $\theta(t)$ and $\dot{\theta}(t)$ are the roll angle and the roll velocity, respectively. I is the virtual or total moment of inertia in roll, B is the damping moment term, C is the restoring moment term and $M(t)$ represents the random wave excitation moment.

The virtual moment of inertia I, generally consists of two parts: the moment of inertia in roll I_{44} and the added mass moment term A_{44}, i.e.

$$I = I_{44} + A_{44} \tag{2}$$

The roll damping normally has three kinds of components: the wave damping due to radiation at the free surface; the damping caused by vortex shedding and flow separation as well as the viscous friction damping. In general, these terms are coupled with each other, hence the quantitative evaluation of the roll damping is difficult. Nevertheless, the linear-plus-quadratic damping (LPQD) model, which has been verified by numerous studies of experimental data, is a good expression used in the SDOF Eq. (1). This model is given as