Nuclear Instruments and Methods in Physics Research B 336 (2014) 31–36

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Parametric gamma-radiation at the anomalous passage conditions

Abbas Ahmadi [a,*], Ilya Feranchuk [b]

[a] Physics Department, Malayer Branch, Islamic Azad University, Malayer, Iran
[b] Belarusian State University, 4 Nezavisimosty Ave., 220030 Minsk, Belarus

ARTICLE INFO

Article history:
Received 11 April 2014
Received in revised form 16 May 2014
Accepted 1 June 2014

Keywords:
Parametric gamma-radiation (PGR)
Dynamical diffraction theory
Anomalous passage
Mössbauer crystal

ABSTRACT

Dynamical diffraction theory of the parametric gamma-radiation (PGR) from relativistic electrons in a thick crystal with Mössbauer nuclei is considered. A detailed analysis of the influence of suppression of photoabsorption for the radiated γ-quanta at an ideal single crystal is presented taking into account both nuclear resonance and electron scattering of the photons. The obtained results allow one to choose the optimal conditions for the observation of the γ-quanta anomalous passage at the Laue case. It is shown that the radiation intensity is drastically increased at such conditions.

1. Introduction

Characteristics of the parametric X-ray radiation (PXR) were investigated in detail in either theory (see the review in [1]) and experiment (see, [2] and the references therein). The role of the dynamical diffraction effects in PXR was considered recently in [3].

Possibility to apply the PXR mechanism to generate resonant γ-rays for the Mössbauer experiments - parametric gamma-radiation (PGR) – was suggested in [4] and specific features of this radiation were analyzed qualitatively. It was shown that the PGR quanta as well as the PXR photons are emitted by a relativistic electron due to diffraction of its electromagnetic field in the crystal with nuclei of the Mössbauer type. More detailed analysis of the PGR characteristics was considered in [5] in the framework of the kinematical diffraction theory taking into account the interference between electron and nucleus resonant parts of a crystal X-ray polarizability. It was shown that the PGR intensity was proportional to the crystal thickness L and could provide spectral brightness of the resonant radiation source comparable with the synchrotron radiation (SR) source after the crystal monochromator [6]. These results are applicable only in the case when the crystal thickness is less than the extinction length [7] $L < L_{ext}$. For the resonant γ-radiation this value has the same order as the absorption length L_{abs} that is rather small $L_{abs} \sim 1$ μm.

However the value L_{abs} increases essentially at the diffraction condition due to the Borrmann effect [7] that leads to increase of the PXR intensity [3]. This effect could be even more important for γ-quanta moving in the Mössbauer crystals. The dynamical diffraction theory for this case was developed in [8]. It was shown there that under certain conditions it is possible to suppress almost fully the inelastic scattering channels in such systems. The suppression of the nuclear reaction occurs under diffraction conditions when the photons impinge on the crystal at the Bragg angle. The crystal then becomes transparent to the resonant quanta, whereas under ordinary conditions a thin layer of the substance completely absorbs these photons [9].

The purpose of the present article is to analyze the characteristics of PGR in the framework of the dynamical diffraction theory in order to find the optimal conditions for increasing of the PGR intensity. We consider here the Mössbauer crystals [10], for which the interaction of the emitted PGR quanta with the nuclei has a resonant character with definite ratio of the inelastic and elastic scattering channels [11]. We analyze the influence of the dynamical diffraction effects on the PGR photon generation in a thick crystal taking into account the interference between the Rayleigh and nuclear scattering of the emitted γ-quanta. A general expression is derived for the integral intensity of the PGR quanta for arbitrary relations between the imaginary and real parts of the crystal polarizability. Numerical results for the crystal with Mössbauer isotope $^{57}_{26}$Fe are also considered.

2. Analysis of the dynamical diffraction effects in the case of PGR

General expression for the spectral-angular distribution of the quanta radiated from the electron in a crystal due to the

* Corresponding author. Tel.: +98 9188515470; fax: +98 8132230195.
E-mail addresses: a.ahmadi@malayeriau.ac.ir, ahmadi.abbas1977@gmail.com (A. Ahmadi).

http://dx.doi.org/10.1016/j.nimb.2014.06.001
0168-583X/

parametric mechanism was considered in many works (see, for example [1]). In the case of the dynamical diffraction theory it was recently discussed in our paper [3]. Therefore we omit the calculation details and start from the following formula for distribution of the photons with energy $\hbar\omega$ and polarization $\vec{e}_s$ near the diffraction peak:

$$\frac{\partial^2 N_s^g}{\partial\vec{\Omega}\partial\omega} = \frac{e_0^2}{\hbar\omega\pi^2 c}(\vec{e}_{sg}\vec{n}_g)^2|\gamma_0|\beta^2\left|\sum_{\mu=1}^{2}\lambda_{\mu s}^g\left[\frac{1}{q_0^g}-\frac{1}{q_{\mu s}^g}\right](1-e^{-ikLq_{\mu s}^g\gamma_0^{-1}})\right|^2,$$
$$k=\frac{\omega}{c}. \quad (1)$$

This peak is directed along the vector $\vec{k}_B$ and other characteristic vectors and angles are shown at Fig. 1:

$$\vec{k}_B = \frac{\vec{v}}{v^2}\omega_B + \vec{g}; \quad \omega_B = c\frac{g^2}{2|g_z|}. \quad (2)$$

Other values in Eq. (1) are defined as follows (the details are in [3]):

$$q_0^{(g)} = \frac{1}{k_B L_0^{(g)}} = \frac{m^2c^4}{E^2} + \theta_{(g)}^2;$$
$$q_{\mu s}^{(g)} = \frac{1}{k_B L_{\mu s}^{(g)}} = \frac{m^2c^4}{E^2} + \theta_{(g)}^2 + \theta_s^2(L) - 2\epsilon_{\mu s};$$
$$\theta_s^2(L) = \frac{E_s^2}{E^2}\frac{L}{L_R}. \quad (3)$$

$$\theta_{(g)}^2 = \vartheta_x^2 + \vartheta_y^2;\ \vec{k} = \vec{k}_B + \vec{u};\ \vartheta_{x,y} = \frac{u_{x,y}}{k_B};\ \gamma_0 = \frac{\vec{k}_B\vec{N}}{k};$$
$$\vec{n}_g = \vec{k}_B/k_B;\ \vec{e}_{\sigma g} = \frac{1}{k_B}[\vec{k}\times\vec{n}_g];\ \vec{e}_{\pi g} = [\vec{e}_{\sigma g}\times\vec{n}_g];\ \beta = \frac{v(\vec{k}_B\vec{N})}{k(\vec{v}\vec{N})}. \quad (4)$$

Here m, E, e_0 and $\vec{v}$ are correspondingly the mass, energy, charge and velocity of the electron, The quantities $k_B q_0^{(g)}$ and $k_B q_{\mu s}^{(g)}$ are the changes of the electron momentum projected to the direction of its velocity in the vacuum and in the crystal, respectively, due to photon emission in the direction $\vec{k}_B = \omega_B\vec{v}/v^2 + \vec{g}$ (the case $\vec{g} = 0$ is included). Thus the inverse values $L_0^{(g)} = (k_B q_0^{(g)})^{-1}$ and $L_{\mu s}^{(g)} = (k_B q_{\mu s}^{(g)})^{-1}$ are the coherent radiation lengths, that defines the length of the electron path in the vacuum and in the crystal which give coherent radiation. The radiation intensity is

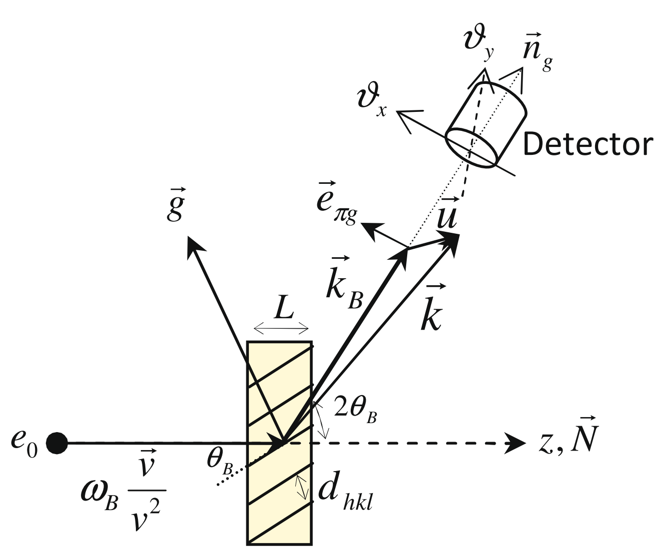

Fig. 1. Characteristic vectors and angles that define the PGR spectral-angular distribution.

proportional to the square of the coherent radiation length and, therefore, the maximum number of photons is emitted in the directions where $L_0^{(g)}$ and $L_{\mu s}^{(g)}$ are maximal [12], $\vec{N}$ is the unit vector of the normal to the entrance crystal surface (Fig. 1). The mean square angle $\theta_s^2(L)$ takes into account approximately the electron multiple scattering in the crystal, where L is the crystal thickness, the universal value $E_s \approx 21$ MeV and L_R is the radiation length depending on the medium characteristics [13].

Other parameters are connected with the quantum dynamical diffraction:

$$\epsilon_{\mu s} = \frac{1}{4}\left\{[\chi_0+\beta\chi_0-\beta\alpha_B]\pm\sqrt{[\chi_0+\beta\chi_0-\beta\alpha_B]^2+4\beta[\chi_0\alpha_B-(\chi_0^2-c_s^2\chi_{\vec{g}}\chi_{-\vec{g}})]}\right\},$$
$$\alpha_B = \frac{2\vec{k}\vec{g}+g^2}{k^2};\quad c_s = (\vec{e}_s\vec{e}_{sg});\ \lambda_{1(2)s}^g = \frac{c_s\chi_g}{2(\epsilon_{2(1)s}-\epsilon_{1(2)s})}. \quad (5)$$

The index s denotes the polarization and μ means that choice of $(\pm)$ corresponding to the sign before the square root in Eq. (5), $c_s = (\vec{e}_s\vec{e}_{sg})$ is the polarization factor, and $\vec{e}_s$ and $\vec{e}_{sg}$ are the unit vectors corresponding to σ $(s=1)$ and π $(s=2)$ polarizations of the incident and diffracted waves, respectively [7]. The dimensionless value α_B is very important for the diffraction theory, in our case it characterizes the deviation of the wave vector of the emitted photon from the exact Bragg condition corresponding to $\alpha_B = 0$. The characteristics of the crystal $\chi_0, \chi_{\vec{g}}$ are the Fourier components of the susceptibility that are directly connected with the amplitudes of the photon coherent scattering by the electrons and nuclei of the crystal. The obvious form of these expressions are well known in the theory of X- and γ-rays diffraction [4] and in the case of the Mössbauer crystal include the electron χ_e and nucleus χ_n parts [5]:

$$\chi_0 = \chi_e(0,\omega_r) + \chi_n(\omega),\ \ \chi_{\vec{g}} = \chi_e(\vec{g},\omega_r) + \chi_n(\omega,\vec{g}) \quad (6)$$

where the electron part in the case of PGR should be calculated at the resonant frequency ω_r and can be considered as a constant and other values are defined by the formulas [5]:

$$\chi_e(\vec{g},\omega_r) = \frac{4\pi S(\vec{g})e^{-W(\vec{g})}}{\Omega_0\omega_r^2}f_e(\vec{g}),\quad f_e(\vec{g}) = -\frac{e_0^2}{mc^2}F_a(\vec{g});$$
$$\chi_n(\omega,\vec{g}) = \frac{4\pi S(\vec{g})e^{-W(\vec{g})}}{\Omega_0\omega_r^2}f_n(\omega),$$
$$f_n(\omega) = -\frac{\eta c}{\omega_r(1+\alpha_c)}\frac{\Gamma/2}{\hbar(\omega-\omega_r)+i\Gamma/2};\quad \chi_0 = \chi_{\vec{g}}|_{\vec{g}=0}. \quad (7)$$

Here Ω_0 is the volume of crystal unit cell; f_e, f_n are the scattering amplitudes of photons with energy $\hbar\omega_r$ by electron shell and nucleus respectively; $S(\vec{g})$ is the structure factor of crystal unit cell; $F_a(\vec{g})$ is the atomic scattering factor $(F(0) = Z_a$ is the atomic charge); $e^{-W(\vec{g})}$ is the Debye–Waller factor; η is the relative concentration of the Mössbauer isotope in the unit cell and α_c is the internal conversion coefficient.

We suppose to consider the PGR intensity for the case of the anomalous decrease of the absorption of the emitted γ-quanta. In accordance with [3] this effect is maximal for σ-polarization and corresponds to the spectral-angular range when the diffraction coherent length $L_{\mu s}^g$ is large in comparison with the vacuum coherent length L_0^g. It leads to simplification of Eq. (1):

$$\frac{\partial^2 N_\sigma^g}{\partial\vec{\Omega}\partial\omega} = \frac{e_0^2}{\hbar\omega\pi^2 c}\left(\vartheta_y^2+\frac{1}{2}\theta_s^2\right)\frac{|\gamma_0|\beta^2|\chi_g|^2}{4|\epsilon_{1\sigma}-\epsilon_{2\sigma}|^2}\left|\sum_{\mu=1}^{2}(-1)^\mu\frac{(1-e^{-ikLq_{\mu\sigma}^g\gamma_0^{-1}})}{q_{\mu\sigma}^g}\right|^2. \quad (8)$$

There are some differences in analysis of this equation for the PXR and PGR. In the former case one could integrate Eq. (8) analytically over the frequency taking into account the inequality $\chi_g'' \ll |\chi_g'|$ for the imaginary and real parts of the susceptibility