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The effective differential cross section for inelastic energy loss of electrons in matter

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ABSTRACT

The effective differential cross section (DCS) for the inelastic energy loss of electrons propagating in matter is proposed that reproduces the stopping power and the straggling of energy loss but provides a total inelastic cross section that is significantly smaller than the one for the real DCS. The number of inelastic collisions of electrons in matter when using the effective DCS in Monte Carlo (MC) simulations is significantly lower than in the case of using the real DCS. The results of our MC simulations of electron propagation in aluminium using the proposed DCS are in good agreement with experimental data and with MC simulations using traditional approaches for description of inelastic energy loss of electrons in matter.

1. Introduction

A detailed MC simulation of electron propagation in matter is widely used for electrons with low initial energies. In the case of high-energy electrons a detailed simulation consumes a lot of computational time due to very high number of electron collisions in matter. Analytical DCSs used for description of elastic [1–3] and inelastic [4–6] collisions of electrons in matter allow one to decrease the computational cost in modeling of a single electron collision but do not cardinally change the situation. Continuous-slowing-down approximation (CSDA) allows one significantly decrease the computational time in modeling of inelastic energy loss [7], but it does not effect on the number of elastic collisions of electrons in matter. The effective DCS for elastic scattering of electrons by atoms developed in [8,9] is aimed at decreasing the computational cost of simulation of elastic collisions. The effective DCS for elastic scattering exactly reproduces known energy dependencies for the first and second transport cross sections but the total elastic cross section for the effective DCS is significantly smaller than the one for the real DCS. That is why the number of elastic collisions of high-energy electrons in matter when using the effective DCS for elastic scattering is significantly lower than in the case of using the real DCS. The combination of the effective DCS for elastic scattering of electrons and the CSDA for the description of inelastic energy losses implemented in the MC strategy of [10] leads to a significant decrease in the total number of collisions experienced by electrons in matter and hence significantly decreases the computational cost of the MC simulation. Results of simulations using the CSDA strategy are quite good but not exactly correspond to detailed MC simulations [10]. A difference of detailed MC simulations and the CSDA is caused by the stochastic nature of electron energy loss in inelastic collisions, not accounted for in the CSDA. It is reasonable to assume that an approach for the description of the inelastic energy loss of electrons in matter that will reproduce not only the stopping power of electrons (as in the CSDA), but also the energy loss straggling should be more precise than the CSDA in MC simulations.

In this work, the effective DCS for inelastic energy loss of electrons will be developed so that the stopping power and the energy loss straggling are reproduced exactly, but the total inelastic cross section for the effective DCS will be significantly smaller than the one for the real DCS. Consequently, the number of inelastic collisions of high-energy electrons in matter when using the effective DCS will be significantly lower than in the case of using the real DCS. The combination of the effective DCS for inelastic energy loss of electrons together with the effective DCS for elastic scattering of electrons from [8,9] give us ability to carry out detailed MC simulations of passage of high-energy electrons through matter with reasonable computational costs.

2. The effective differential cross section for inelastic energy loss

The total cross section of inelastic collisions σ_{in}, stopping power S_{in} and the energy straggling parameter Ω^2_{in} can be defined through

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a DCS for electron energy loss in inelastic collisions $d\sigma_{in}/dW$ by the following equations:

$$\begin{aligned}\sigma_{in} &= \int_0^{W_{max}} \frac{d\sigma_{in}}{dW} dW \\ S_{in} &= N\cdot\int_0^{W_{max}} W\frac{d\sigma_{in}}{dW} dW \\ \Omega_{in}^2 &= N\cdot\int_0^{W_{max}} W^2\frac{d\sigma_{in}}{dW} dW\end{aligned}, \qquad (1)$$

where W is the energy loss, W_{max} is the maximal limit of the electron energy loss in inelastic collision and N is the number of atoms in the medium per unit volume. In our calculations we use $W_{max} = E/2$, where E is the energy of the electron.

The energy loss of an electron passing through matter is frequently defined using simple approaches. Thus, for example in [4,5,9] the DCS $d\sigma_{in}/dW = A/W^2$, $W_{min} \leqslant W \leqslant W_{max}$, where W_{min} is the minimal limit of the electron energy loss, was used in MC simulations. Let us introduce the effective DCS for electron energy loss in inelastic collisions in the following form:

$$\frac{d\sigma_{in}^{eff}}{dW} = \frac{A}{W^n}, W_{min} \leqslant W \leqslant W_{max}. \qquad (2)$$

The parameters A and W_{min} of the effective DCS (2) will be defined in such a way that the stopping power and the energy straggling parameter are reproduced exactly. The parameter n will be chosen to provide a total cross section for the effective DCS (2) smaller than σ_{in}. The total cross section, stopping power and the energy straggling parameter for the effective DCS (2) will be designated as σ_{in}^{eff}, S_{in}^{eff} and $\Omega_{in,eff}^2$ correspondingly. Substituting the effective DCS (2) instead of $d\sigma_{in}/dW$ into Eq. (1) we obtain the following expressions for the effective parameters:

$$\begin{aligned}\sigma_{in}^{eff} &= \frac{A}{n-1}\cdot\left(\left(\frac{1}{W_{min}}\right)^{n-1} - \left(\frac{1}{W_{max}}\right)^{n-1}\right) \\ S_{in}^{eff} &= N\cdot\frac{A}{n-2}\cdot\left(\left(\frac{1}{W_{min}}\right)^{n-2} - \left(\frac{1}{W_{max}}\right)^{n-2}\right) \\ \Omega_{in,eff}^2 &= N\cdot\frac{A}{n-3}\cdot\left(\left(\frac{1}{W_{min}}\right)^{n-3} - \left(\frac{1}{W_{max}}\right)^{n-3}\right)\end{aligned}. \qquad (3)$$

In order to define right-hand sides of Eq. (3) in the cases $n = 1, 2, 3$ we can use the following limit: $\lim_{z\to 0}\left((1/W_{min})^z - (1/W_{max})^z\right)/z = \ln(W_{max}/W_{min})$. Parameters A and W_{min} of the effective DCS (2) at any chosen value of the parameter n should be derived from the following equations:

$$S_{in}^{eff} = S_{in}, \Omega_{in,eff}^2 = \Omega_{in}^2. \qquad (4)$$

Using Eq. (3) and the equation $\Omega_{in,eff}^2/S_{in}^{eff} = \Omega_{in}^2/S_{in}$ composed from Eq. (4) we arrive at the following equation

$$\frac{w^{3-n}-1}{w^{2-n}-1}\cdot\frac{n-2}{n-3} = \frac{\Omega_{in}^2}{S_{in}W_{max}}. \qquad (5)$$

where $w = W_{min}/W_{max}$. Solving Eq. (5) we derive the parameter w and thus $W_{min} = w \cdot W_{max}$. After that, using the equation $S_{in}^{eff} = S_{in}$ and Eq. (3) we can define the parameter A of the effective DCS (2) in the following form:

$$A = \frac{W_{max}^{n-2}\cdot(n-2)}{(w^{2-n}-1)}\cdot\frac{S_{in}}{N} \qquad (6)$$

Parameters A and $W_{min} = w \cdot W_{max}$ derived from Eqs. (5) and (6) define the effective DCS (2) through S_{in} and Ω_{in}^2. The effective inelastic cross section can be obtained from Eqs. (3) and (6) in the following form:

$$\sigma_{in}^{eff} = \frac{S_{in}}{NW_{max}}\cdot\frac{n-2}{n-1}\cdot\frac{w^{1-n}-1}{w^{2-n}-1}. \qquad (7)$$

Let us determine the effective DCS (2) for inelastic energy loss of electrons in Al. The parameter $\Omega_{in}^2/(S_{in}W_{max})$ that defines the right-hand side of Eq. (5) is shown in Fig. 1 as a function of the electron energy. The straggling parameter Ω_{in}^2 calculated through the DCS $d\sigma_{in}/dW$ from [9] and the stopping power $S_{in}(E)$ from [11,12] are used for the calculation of the parameter $\Omega_{in}^2/(S_{in}W_{max})$ for Al. Fig. 2 shows the effective DCS (2) normalised to S_{in}/NW_{max}^2 for 1 MeV electrons. The parameters W_{min} and A that define the effective DCS are derived from Eqs. (5) and (6) at corresponding values of the parameter n. It is seen from Fig. 2 that the increase of the parameter n leads to increasing the parameter W_{min} of the effective DCS (2).

The energy dependency of the effective inelastic cross section $\sigma_{in}^{eff}(E)$ for electrons in Al is shown in Fig. 3 at different values of the parameter n. It is seen from Fig. 3 that the parameter n increase

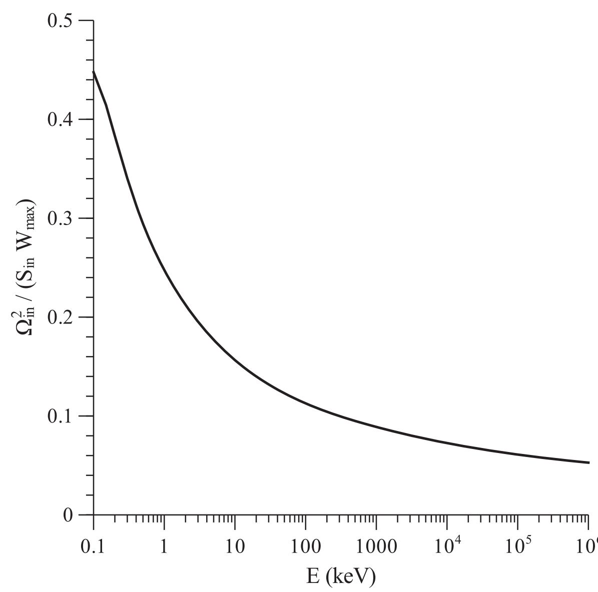

Fig. 1. The parameter $\Omega_{in}^2/(S_{in}W_{max})$ for electrons in Al calculated using the DCS $d\sigma_{in}/dW$ from [9].

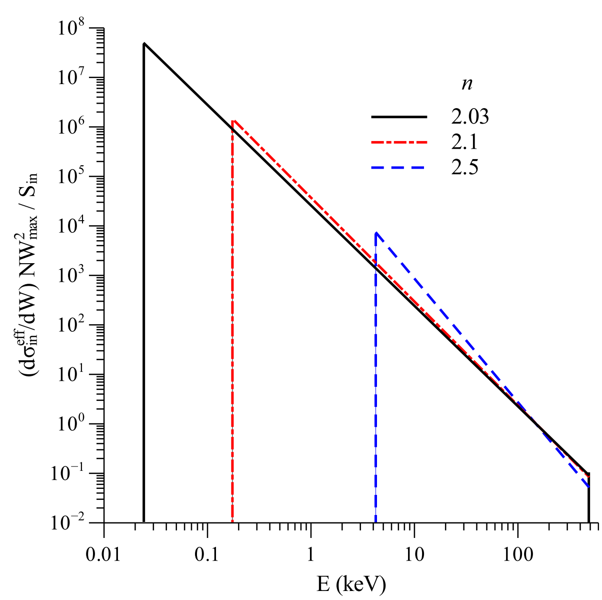

Fig. 2. The effective DCS for inelastic energy loss of 1 MeV electrons in Al at different values of the parameter n shown near the curves.