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Parameter estimation with correlated outputs using fidelity maps

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ABSTRACT

This paper introduces a new approach for parameter estimation and model update based on the notion of fidelity maps. Fidelity maps refer to the regions of the parameter space within which the discrepancy between computational and experimental data is below a user-defined threshold. It is shown that fidelity maps provide an efficient and rigorous approach to approximate likelihoods in the context of Bayesian update or maximum likelihood estimation. Fidelity maps are constructed explicitly in terms of the parameters and aleatory uncertainties using a Support Vector Machine (SVM) classifier. The approach has the advantage of handling numerous correlated responses, possibly discontinuous, without any assumption on the correlation structure. The construction of accurate fidelity map boundaries at a moderate computational cost is made possible through a dedicated adaptive sampling scheme. A simply supported plate with uncertainties in the boundary conditions is used to demonstrate the methodology. In this example, the construction of the fidelity map is based on several natural frequencies and mode shapes to be matched simultaneously. Various statistical estimators are derived from the map.

1. Introduction

Computational models are used, for instance, to predict the static or dynamic behavior of structures. However, there might be marked discrepancies between the prediction of the model and experimental data. In order to reduce this difference, the model needs to be calibrated (or updated) by searching parameter values (e.g., material properties) that best "match" the data. For example, in modal analysis, the characteristics of the model (e.g., stiffness and mass distribution) will be modified so as to match experimental natural frequencies and mode shapes [1].

In engineering applications, the most widely used technique is the least square approach. However, uncertainties might have a pronounced effect on the responses of the system and this approach, often implemented in a deterministic way, is in general not suitable [2,3]. For this reason, statistical approaches have been favored to extract distributions of update parameters and responses. The two most common statistical approaches are the maximum likelihood estimate and Bayesian update. While the maximum likelihood approach [4,5] finds the most "probable" values of the parameters to be estimated, the Bayesian method [6,7] focuses on refining the parameter distributions inferred from previous knowledge.

At the core of both approaches, lies the computation of the likelihood. In most engineering applications, the likelihood is difficult to compute and is approximated using assumptions on the correlation structure of the responses (e.g., independence). This difficulty is further exacerbated by computationally intense simulations, large number of responses [8,9], and discontinuous responses.

The proposed update approach is designed to provide a flexible scheme which tackles the aforementioned difficulties. This is done through the identification of the regions of the parameter space where the discrepancy between model and experimental outputs is below a given threshold. These regions form a "fidelity map" and can be shown to provide a rigorous and efficient approximation of the likelihood without restrictive assumptions (Section 2 provides an illustrative example of a fidelity map).

The boundaries of the fidelity maps are constructed using a Support Vector Machine (SVM) which is a classification technique. It is used to explicitly separate data belonging to two classes [10–13]. In the context of the fidelity maps, this binary classification is performed based on the discrepancy between computational and experimental data which is either smaller or larger than a given user-defined threshold. In order to obtain an accurate fidelity map using a reasonable number of simulation calls, the SVM boundary is refined using an adaptive sampling scheme [14]. That is, most of the computational cost is concentrated in the construction of the fidelity maps. The likelihood can then be efficiently obtained as a sub-product of the fidelity map.

Many frameworks for model update have been developed. Of particular importance and impact is the work by Kennedy and

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O'Hagan [15] which has become a reference in the domain. For instance, the use of a Gaussian process was initially introduced in this work. This approach was subsequently used in other strategies [16]. There is also a large body of literature dedicated to Bayesian approaches [17,18]. The proposed work sets itself apart from existing approaches by enabling model updating for problems with numerous responses (potentially discontinuous) without any a-priori on the correlation structure which is implicitly accounted for during the construction of the fidelity map. This flexibility stems from the use of a classification technique such as SVM for the construction of the fidelity map boundaries.

This paper is constructed as follows. Section 2 provides the notation and the main concepts of the proposed approach. Section 3 presents the statistical estimators used. Section 4 describes the fidelity map and the approximation of the likelihood. Section 4.1 provides a background on SVM classifiers. Section 4.2 describes the adaptive sampling scheme used to accurately build the fidelity maps. Finally, Section 5 provides results on a demonstrative example consisting of a plate with uncertainty in the boundary conditions. In the example, several frequencies and mode shapes are to be matched simultaneously. For the sake of completeness, the results are compared to approaches where the responses are assumed independent or if a residual, which encompasses all the responses within one quantity, is used.

2. Illustrative example and notations

Consider the responses $\mathbf{y}$ of a model and the corresponding experimental measurements $\mathbf{y}^{exp}$. The responses of the system are governed by two types of parameters: the first set are the parameters to estimate $\mathbf{x}$ (e.g., material properties) while the second one, $\mathbf{A}$, are the “aleatory” parameters which are not to be estimated but introduce uncertainty (e.g., external load). The probability density function (PDF) of a random variable X is noted $\mathbf{f}_X$ and its cumulative distribution function (CDF) is noted $\mathbf{F}_X$.

As an illustrative example, consider a model in the form of a cantilever column (e.g., representative of a building) subjected to wind loading (Fig. 1(a)). In this academic example, we wish to estimate the bending stiffness of the column $K \equiv x$ based on a set of experimental data $\boldsymbol{y}^{exp}$ (e.g., deflection δ) knowing that the column is subjected to a random load $F \equiv A$ with known probabilistic distribution.

Fig. 1(b) depicts the construction of the fidelity map corresponding to p experiments and n responses (e.g., displacements, accelerations, etc.) per experiment. The fidelity map defines the region of the space where the relative discrepancy between model and experiments Δ_{fi} is lower than 1% for every response. It is accurately constructed with a Support Vector Machine classifier and an adaptive sampling scheme described in Section 4. The fidelity map is then used to approximate the likelihood (Fig. 2) which allows one to update the model through maximum likelihood or Bayesian update. For the reader who is not familiar with these statistical estimators, they are described in the following section which also underlines the advantages of using a fidelity map for their computation.

3. Background

3.1. Maximum likelihood estimate

Maximum likelihood estimates (MLE) were originally designed for the statistical inference of hyper-parameters of distributions:

$$\boldsymbol{\theta}^{MLE} = \underset{\boldsymbol{\theta}}{\mathbf{argmax}} \prod_{i=1}^{n_v} \mathbf{f}_X(x_i|\boldsymbol{\theta}) \tag{1}$$

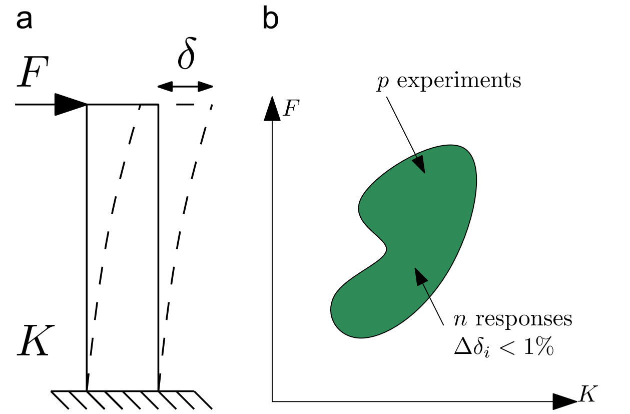

Fig. 1. Illustrative example. Calibration of the stiffness K of a column subjected to a random (aleatory) load F based on experimental responses. (a) Model and (b) fidelity map.

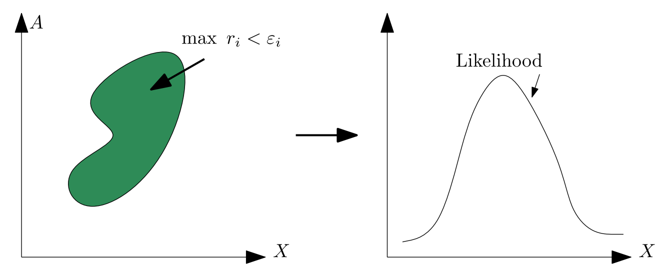

Fig. 2. The fidelity map is then used to build an approximation of the likelihood (up to a constant).

where $\mathbf{x} = [x_1, \ldots, x_{n_v}]$ are n_v i.i.d observations of a random variable X following a PDF $\mathbf{f}_X(x|\boldsymbol{\theta})$ of hyper-parameters $\boldsymbol{\theta}$. This notion can be extended to engineering applications by considering that some output responses $\mathbf{y}$ follow a joint PDF where uncertainties are due to $\mathbf{A}$ and parametrized by $\mathbf{x}$ (i.e., $\mathbf{f}_{\mathbf{y}(\mathbf{x},\mathbf{A})}(\mathbf{y}|\mathbf{x})$). Therefore, the maximum likelihood estimate for parameter identification reads:

$$\mathbf{x}^{MLE} = \underset{\mathbf{x}}{\mathbf{argmax}} \prod_{i=1}^{p} \mathbf{f}_{\mathbf{y}(\mathbf{x},\mathbf{A})}(\mathbf{y}^{exp,(i)}|\mathbf{x}) \tag{2}$$

where $\mathbf{y}^{exp,(i)}$ are the ith experimental set of n responses. In the case of a single set of measurements ($p=1$), as used in this work, Eq. (2) becomes:

$$\mathbf{x}^{MLE} = \underset{\mathbf{x}}{\mathbf{argmax}} \mathbf{f}_{\mathbf{y}(\mathbf{x},\mathbf{A})}(\mathbf{y}^{exp}|\mathbf{x}) \tag{3}$$

3.2. Bayesian estimate

While MLE follows a frequentist approach, and considers $\mathbf{x}$ as deterministic, Bayesian updating considers $\mathbf{X}$ as random variables. Bayesian estimators are derived from the Bayes formula:

$$\mathbf{f}_{A|B}\mathbf{f}_B = \mathbf{f}_{B|A}\mathbf{f}_A$$

Specializing it to engineering applications:

$$\mathbf{f}_{\mathbf{X}}(\mathbf{x}|\mathbf{y}^{exp}) = \frac{\mathbf{f}_{\mathbf{y}(\mathbf{x},\mathbf{A})}(\mathbf{y}^{exp}|\mathbf{x})\mathbf{f}_{\mathbf{X}}(\mathbf{x})}{\mathbf{f}_{\mathbf{y}(\mathbf{X},\mathbf{A})}(\mathbf{y}^{exp})} \tag{4}$$

where:

(i) $\mathbf{f}_{\mathbf{X}}(\mathbf{x}|\mathbf{y}^{exp})$ is the posterior distribution;
(ii) $\mathbf{f}_{\mathbf{y}(\mathbf{x},\mathbf{A})}(\mathbf{y}^{exp}|\mathbf{x})$ is the likelihood;
(iii) $\mathbf{f}_{\mathbf{X}}$ is the prior distribution;
(iv) $\mathbf{f}_{\mathbf{y}(\mathbf{X},\mathbf{A})}(\mathbf{y}^{exp})$ is a normalizing constant which represents the total probability density to observe $\mathbf{y}^{exp}$.