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Optimal design of a novel tuned mass-damper–inerter (TMDI) passive vibration control configuration for stochastically support-excited structural systems

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ABSTRACT

This paper proposes a novel passive vibration control configuration, namely the tuned mass-damper–inerter (TMDI), introduced as a generalization of the classical tuned mass-damper (TMD), to suppress the oscillatory motion of stochastically support excited mechanical cascaded (chain-like) systems. The TMDI takes advantage of the "mass amplification effect" of the inerter, a two-terminal flywheel device developing resisting forces proportional to the relative acceleration of its terminals, to achieve enhanced performance compared to the classical TMD. Specifically, it is analytically shown that optimally designed TMDI outperforms the classical TMD in minimizing the displacement variance of undamped single-degree-of-freedom (SDOF) white-noise excited primary structures. For this particular case, optimal TMDI parameters are derived in closed-form as functions of the TMD mass and the inerter constant. Furthermore, pertinent numerical data are furnished, derived by means of a numerical optimization procedure, for a 3-DOF classically damped primary structure base excited by stationary colored noise, which exemplify the effectiveness of the TMDI over the classical TMD to suppress the fundamental mode of vibration for MDOF structures. It is concluded that the incorporation of the inerter in the proposed TMDI configuration can either replace part of the TMD vibrating mass to achieve lightweight passive vibration control solutions, or improve the performance of the classical TMD for a given TMD mass.

1. Introduction

The idea of attaching an additional free-to-vibrate mass to dynamically excited structural systems (primary structures) to suppress their oscillatory motion is historically among the first passive vibration control strategies in the area of structural dynamics [1–4]. This idea relies on designing or "tuning" the mechanical devices that link the added mass to the primary structure to achieve a "resonant" out-of-phase motion of the mass. In this context, Frahm [1] introduced the use of a linear spring-mass attachment to suppress the oscillations of harmonically excited primary structural systems in naval and mechanical engineering applications. This early "dynamic vibration absorber" was able to reduce the oscillations of single-degree-of-freedom (SDOF) primary structures within a narrow range centered at a particular (pre-specified) frequency of excitation. Later, Ormondroyd and Den Hartog [2] enhanced the effectiveness of the above absorber to dissipate the kinetic energy of primary structures subject to harmonic excitations by appending a viscous damper (dashpot) in parallel to the linear spring. Further, a semi-empirical "optimum" design procedure has been established by Den Hartog [3] and Brock [4] to "tune" the damping and stiffness properties for an *a priori* specified mass of this spring-mass-damper attachment such that the peak displacement of harmonically excited undamped SDOF primary structures is minimized (see also [5]). This design/tuning procedure relies on the "fixed point" assumption which states that all frequency response curves of the resulting two-DOF dynamical system pass through two specific points; the location of these points being independent of the damping coefficient of the dashpot. Thus the tuned spring-mass-damper attachment, commonly termed in the literature as the "tuned mass-damper" (TMD), achieves the suppression of the oscillatory motion of harmonically excited primary structures over a wider range of exciting frequencies compared to a spring-mass attachment. Recently, the fixed point-based tuning procedure was shown to be very close to the "exact" solution for the optimal tuning of the classical TMD [6].

Although alternative arrangements of linear springs and dashpots (viscous dampers) have been considered in the literature to attach a mass to primary structures (see e.g., [7,8] and references therein), the above discussed "classical" TMD configuration (mass

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attached via a spring and a dashpot in parallel) is the most widely studied in the literature and the most commonly used one for passive vibration control of various mechanical and civil engineering structures and structural components. In particular, motivated mostly by earthquake engineering applications, substantial research work has been devoted to investigate the potential of using the classical TMD to mitigate the motion of stochastically support-excited primary structures. Using standard analytical techniques, optimal TMD parameters (damping and stiffness coefficients of the linking spring-damper elements) can be readily obtained in closed-form as functions of the TMD mass to minimize the response variance of undamped SDOF primary structures subject to white noise support excitation [9,10]. However, for the case of damped SDOF primary structures subjected to stochastic support excitations, the derivation of optimal TMD parameters by analytical approaches becomes a challenging task [11]. To this end, numerical optimization techniques are commonly employed for optimum design of TMDs to minimize the response variance for such primary structures (see e.g., [12–15]). Alternatively, simplified approximate solutions for the problem at hand have been reached by making the assumption of "lightly" damped primary structures (e.g., [16,17]). Along similar lines, several researchers proposed different approximate simplified and numerical methods for the design of TMDs for damped linear multi-degree-of-freedom (MDOF) primary structures under stochastic base excitation widely used to model seismically excited multi-storey building structures (see e.g., [18–21] and references therein).

In recent years, several different strategies have been employed to enhance the performance of the classical TMD for passive vibration suppression of stochastically support excited structural systems including the use of multiple classical TMDs (see e.g., [22,23] and references therein), the incorporation of non-linear viscous dampers to the classical TMD configuration [24], and the consideration of hysteretic TMDs (see e.g., [25]). These strategies do offer enhanced performance compared to the classical TMD, however, optimum design/tuning becomes a challenging and computationally involved task, especially for damped MDOF primary structures. Furthermore, analytical and numerical results reported in the extensive relevant literature suggest that the effectiveness of the TMD for vibration mitigation of base-excited structures increases by increasing the attached TMD mass. This is particularly the case for high intensity support excitations (e.g., [13,21]).

In this regard, this paper proposes an alternative passive vibration control solution considering the use of a recently developed two-terminal flywheel (TTF) mechanical device, dubbed the "inerter" by Smith [26], in conjunction with the classical TMD configuration. In theory, the "ideal" inerter is a linear device with two terminals free to move independently which develops an internal (resisting) force *proportional to the relative acceleration of its terminals*. Employing rack and pinion gearing arrangements to drive a rotating flywheel, certain inerter/TTF prototypes have been physically built [26–28]. In fact, inerter/TTF devices are currently used for vibration control of suspension systems in high performance vehicles [29–30]. Further, the performance of various passive vibration control configurations for support excited building structures employing inerters placed in between the ground and the superstructure in a "base isolation" type of arrangement has been studied by Wang et al. [31,32]. It has been established that inerter devices are effective in controlling the response of rigid superstructures exposed to vertical band-limited white noise ground motions. Furthermore, passive vibration control systems comprising inerters in conjunction with springs and dampers have been considered by Lazar et al. [33] for vibration isolation of SDOF and of two-DOF primary systems subjected to recorded earthquake excitations applied along the vertical direction.

The present research work is motivated by the fact that an inerter/TTF device with approximately 1 kg of physical mass can have a constant of resisting force within the range of 60–200 kg depending on the size of the flywheel [27]. Thus, the aim of the herein proposed tuned mass-damper–inerter (TMDI) configuration is to exploit the mass amplification effect of the inerter. Attention is focused on introducing the underlying equations of motion for linear SDOF and MDOF primary structures, to demonstrate that the TMDI constitutes a generalization of the classical TMD and to provide analytical and numerical evidence demonstrating its enhanced performance compared to the TMD. The remainder of this paper is organized as follows: Section 2 introduces the TMDI for the case of linear SDOF primary structures exposed to stochastic support excitation. The governing equations of motion are derived for damped primary structures and analytical expressions for optimum TMDI parameters minimizing the displacement variance for the special case of undamped white noise excited SDOF primary structures are obtained. Section 3 proposes a TMDI configuration to suppress oscillations following the fundamental mode of vibration of support-excited damped MDOF chain-like primary structures. A numerical optimization procedure for optimum design of the TMDI system for these primary structures is also discussed. Section 4 provides numerical data to demonstrate the effectiveness and applicability of the TMDI vis-à-vis the classical TMD for classically damped MDOF chain-like primary structures. Section 5, summarizes the main conclusions of this work.

2. Proposed tuned mass-damper–inerter (TMDI) configuration for single-degree-of-freedom (SDOF) support-excited primary structures

Consider a linear damped single-degree-of-freedom (SDOF) dynamical system (primary structure) modeled by a linear spring of stiffness k_1, a mass m_1, and a viscous damper with damping coefficient c_1, based-excited by an acceleration stochastic process $a_g(t)$. To suppress the oscillatory motion of this primary structure it is herein proposed to consider the classical tuned mass-damper (TMD), in conjunction with a two terminal flywheel (inerter) device as shown in Fig. 1. The TMD consists of a mass m_{TMD} attached to the primary structure via a linear spring of stiffness k_{TMD} and a viscous damper with damping coefficient c_{TMD}. The inerter device connects the TMD mass to the supporting ground. It is noted in passing that the idea of placing the damper in between the TMD mass and the ground instead of in between the TMD mass and the primary structure has been explored in the literature (e.g., [34]). However, such "non-traditional" TMD topologies are not considered in this work.

In Fig. 1, the inerter is depicted by a hatched box which should be interpreted as a mechanical two-terminal device similarly to springs and dampers. To facilitate this interpretation, Fig. 2 depicts

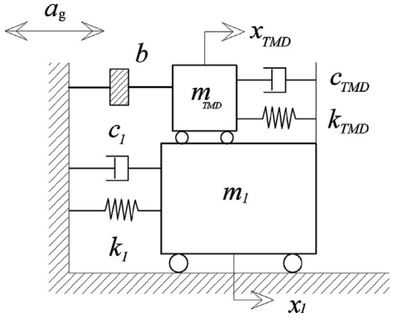

Fig. 1. Single-degree-of-freedom (SDOF) primary structure incorporating the proposed tuned mass-damper–inerter (TMDI) configuration.