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Stability, control and reliability of a ship crane payload motion

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ABSTRACT

This paper investigates the stochastic dynamics, stability and control of a ship-based crane payload motion, as well as the first time passage type of failure. The simplified nonlinear model of the payload motion is considered, where the excitation of a suspension point is imposed due to the heaving motion of waves. The latter enters the system parametrically, leading to a Mathieu type nonlinear equation. The stability boundaries are numerically calculated, using the Lyapunov exponent approach. The control strategy, based on the feedback bang–bang control policy, is implemented to minimize the load's swinging motion. Finally, the first time passage problem is addressed employing Monte-Carlo sampling of the failure process.

1. Introduction

Different types of cranes are used for lifting heavy payloads every day. There are cranes, which operate worldwide today, among which one can name the most commonly used: a rotary crane, boom crane and gantry crane [1]. Besides the issues of reliability and safety of the cranes there are other problems related to their performance such as the maximum payload and how fast they can move the payload from one location to another. The latter may be a critical problem especially at a ship recovery operation, when a large ship has to lift up a small boat or submersible vehicle from the sea surface, despite sea conditions. It is well known that the load, which is transported, may swing due to the motion of the crane or severe atmospheric conditions, especially wind. This motion, if not controlled, may lead to additional forces in the cable, causing the crane to lose its payload or collapse. Obviously, offshore or ship based cranes have an additional source of excitation due to a wave motion. In particular, heaving motion of waves may provoke parametric excitation of the crane base, which can be especially dangerous when the wave frequency is twice larger than the natural frequency of the system. In that case one may expect parametric resonance to occur, which leads to the instability of the payload motion. Results of numerical modeling and experiments of floating cranes behavior may be found, for instance in [2]. No doubt that the excessive amplitude of oscillations, at ship recovery operations, may cause a collision of the payload against the ship with unpredictable consequences. Thus, it is important to understand the behavior of the payload under a wave like motion of an offshore structure in order to develop a proper control strategy. Some control strategies have been proposed earlier for deterministic excitations [1,3,4]. A control strategy for reducing swing oscillations due to stochastic excitation may be found in [5,6].

There are several ways to model a load motion, one of which reduces to a lumped mass system, the motion of which can be represented by a spherical pendulum. Such a system with a constant pendulum length is described by two differential equations in 3D space. There are a number of papers devoted to the dynamics of a spherical pendulum [7–10]. In these papers the authors were concerned with stability boundaries of the system response due to deterministic excitation. The stability analysis was performed on the system, where the sinus type nonlinearity was replaced by a few terms of the Taylor series.

In this paper the authors consider the simplified dynamics of a ship-based crane payload motion, which is governed by a nonlinear Mathieu equation with a narrow band parametric excitation, whereas the motion of a payload is eventually modeled by a planar pendulum. The excitation is considered in the vertical direction due to heaving motion of waves. The waves are modeled using a harmonic function with random phase modulations and the stability boundaries are calculated using the Largest Lyapunov Exponents (LLE). Furthermore, in order to improve the system performance a control of the length of the linearized pendulum is implemented and its influence on the stability boundaries is sought, indicating the asymptotic stability of the bottom equilibrium point. Finally, the first time passage reliability problem is

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considered for different threshold angles for the nonlinear system to quantify the instability rate.

2. Dynamics of a spherical pendulum

Let us consider the dynamics of a spherical pendulum, the suspension point of which is excited in the vertical z direction. In the following derivations we assume a non-stretchable cable of a constant length L. Thus, the coordinates of the pendulum may be expressed as a function of their angles as

$$x = L\sin\theta\sin\phi y = L\sin\theta\cos\phi z = \eta + L\cos\theta \tag{2.1}$$

where $\eta = \eta(t)$ is the displacement of the suspension point (or rigidly connected ship crane). To construct the Lagrangian, it is required to obtain the velocity of the mass M. Let us differentiate Eq. (2.1) with respect to time:

$$\begin{aligned} \dot{x} &= L\dot{\theta}\cos\theta\sin\phi + L\dot{\phi}\sin\theta\cos\phi\dot{y} \\ &= L\dot{\theta}\cos\theta\cos\phi - L\dot{\phi}\sin\theta\sin\phi\dot{z} \\ &= \dot{\eta} - L\dot{\theta}\sin\theta \end{aligned} \tag{2.2}$$

so that the velocity can be written as

$$v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 = \gamma_0^2 + \dot{\eta}^2 - 2L\dot{\eta}\dot{\theta}\sin\theta \tag{2.3}$$

where

$$\gamma_0^2 = L^2\dot{\theta}^2 + L^2\dot{\phi}^2\sin^2\theta \tag{2.4}$$

corresponds to the spherical pendulum motion with a stationary suspension point. Thus, one gets the following expression for the Lagrangian:

$$\Gamma = \tfrac{1}{2}Mv^2 - MgL(1 - \cos\theta) \tag{2.5}$$

Two equations of motion can be obtained using this approach:

$$\frac{d}{dt}\left(\frac{\partial\Gamma}{\partial\dot{\theta}}\right) - \frac{\partial\Gamma}{\partial\theta} = 0\frac{d}{dt}\left(\frac{\partial\Gamma}{\partial\dot{\phi}}\right) - \frac{\partial\Gamma}{\partial\phi} = 0 \tag{2.6}$$

Substituting Eqs. (2.4) and (2.5) into the second equation reduces (2.6) to the following expression:

$$\dot{\phi}\sin^2\theta = H - const \tag{2.7}$$

which can be interpreted as a conservation of angular momentum. The first equation will be written as

$$ML^2\ddot{\theta} - M\ddot{\eta}L\sin\theta + MgL\sin\theta - ML\dot{\phi}^2\sin\theta\cos\theta = 0 \tag{2.8}$$

This equation is the exact equation of the motion of a spherical pendulum with vertically excited suspension point. Simplifying this expression, taking into account the expression for $\dot{\phi}$, one obtains

$$\ddot{\theta} + \left(\Omega^2 - \frac{\ddot{\eta}}{L}\right)\sin\theta - \frac{H^2\cos\theta}{L\sin^3\theta} = 0 \tag{2.9}$$

with $\Omega = \sqrt{g/L}$ denoting the natural frequency. It should be noted that Eq. (2.7) (and therefore Eq. (2.9)) is valid for any positive value of H only when the angle $\theta \neq 0$ or $\theta \neq \pi$, whereas at these equilibrium points the velocity $\dot{\phi}$ becomes infinite. Thus, since this paper is focused on studying the oscillatory motion in θ direction, and not in the conical type motion, zero velocity $\dot{\phi} = 0$ is taken, so that the last term will disappear and equation of motion will resemble the in-plane oscillations of a parametrically excited pendulum. Rotational stochastic potential of a plane pendulum has already been studied in [11].

3. Dynamics of a SDOF model

3.1. Problem statement

Since Eq. (2.9) is nonlinear it is difficult to apply any analytical techniques to solve it approximately. Thus it is required to implement the Taylor expansion and keep a few first terms in the equation. In Cartesian coordinates (x,y), Eq. (2.9) can be written so that [1,12]

$$\ddot{x} + 2\alpha\dot{x} + \Omega^2x + xg(x, y) = -\frac{x}{L}\ddot{\eta} \tag{3.1a}$$

$$\ddot{y} + 2\alpha\dot{y} + \Omega^2y + yg(x, y) = -\frac{y}{L}\ddot{\eta} \tag{3.1b}$$

where α is the viscous damping coefficient, $\eta(t)$ is the exciting force, acting in vertical direction and

$$g(x, y) = \frac{\Omega^2}{2L^2}\left(x^2 + y^2\right) + \frac{1}{L^2}\left(\dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y}\right) \tag{3.2}$$

To simplify the problem further we assume a planar motion, setting out-of-plane motion to zero:

$$\ddot{x} + 2\alpha\dot{x} + \Omega^2x + xg(x, 0) = -\frac{x}{L}\ddot{\eta} \tag{3.3}$$

Eq. (3.3) is a nonlinear equation, which contains the parametric excitation. In the case of a purely periodic excitation Eq. (3.3) will become nonlinear Mathieu type equation. Some numerical results of system (3.3) under a deterministic excitation are reported in [2]. However, in the case of a ship crane the load due to sea waves is narrow-banded and described by a Pierson–Moskowitz (PM) spectra [13]. The latter can be reasonably well modeled by a harmonic function with random phase fluctuations [14,15]:

$$\ddot{\eta} = -\lambda\omega^2\cos q(t), \quad \dot{q} = \omega + \sigma\zeta(t) \tag{3.4}$$

where $\zeta(t)$ is a Gaussian white noise with $\langle\zeta(t)\zeta(t+\tau)\rangle = D\delta(\tau) = \sigma^2\delta(\tau)$ and λ, ω the excitation's amplitude and frequency respectively.

3.2. Stochastic averaging

Since the excitation acting onto the system is multiplicative, it is important to investigate possible parametric instability, which happens when the excitation frequency is twice the natural frequency of the system. Let us introduce slowly varying amplitude and phase of the response:

$$x = aL\cos\Theta(t)\dot{x} = -aL\frac{\omega}{2}\sin\Theta(t)\dot{\Theta}(t) = \frac{\omega}{2} - \dot{\phi} \tag{3.5}$$

Because $g(x, 0)$ contains the second derivative, Eq. (3.3) can be rearranged as follows:

$$\left(\ddot{x} + \Omega^2x\right)\left[1 + \frac{x^2}{L^2}\right] = -2\alpha\dot{x} - \frac{x}{L}\ddot{\eta} + \frac{\Omega^2x^3}{2L^2} - \frac{x\dot{x}^2}{L^2} \tag{3.6}$$

Applying the stochastic averaging technique to Eq. (3.6) one can arrive to a set of two first order nonlinear stochastic differential equations for the response amplitude:

$$\dot{a} = -\frac{2\alpha}{a}\left(\sqrt{1 + a^2} - 1\right) - \frac{4\lambda\Omega}{La^3}\left[2 + a^2 - 2\sqrt{1 + a^2}\right]\sin 2\phi \tag{3.7}$$