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Extension of the regulated stochastic linearization to beam vibrations

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ABSTRACT

In this study, a version of the regulated stochastic linearization technique is proposed for the nonlinear random vibrations of Bernoulli–Euler nonlinear beams. For analysis, in order to balance the error of linearization, we utilize the regulated technique; namely, the appearing nonlinear terms are first replaced by higher-order nonlinear expressions that are subsequently reduced, in stages, to linear ones. It is demonstrated that this seemingly a "roundabout" way is extremely effective to derive a solution that turns out to be much closer to the results provided by the Monte Carlo simulation than those derived via the conventional or potential energy linearization techniques, in the cases of large nonlinearity.

1. Introduction

In 1995, Anh and Di Paola [1] suggested a version of the stochastic linearization technique for solving stochastic nonlinear problems. The authors resorted to introduction of the auxiliary step prior to solving the nonlinear stochastic differential equation. This additional step consists in replacing nonlinear terms with "even more nonlinear" terms. These higher order terms are then replaced by linear term in several steps, in each of them reducing the level of nonlinearity.

This technique called as "regulated Gaussian equivalent linearization" (RGEL) was shown to provide an effective method of solution for single-degree-of-freedom systems [1]. The authors conducted a single-step regulation, implying introduction of the one-step complication of the nonlinear term. Elishakoff et al. [2] extended the methodology of Anh and Di Paola to two-step regulation. The latter extension showed considerable improvement of the results in comparison with both the classical scheme of the stochastic linearization, as well as the single-step regulation, in the Lutes and Sarkani oscillator [3].

Until now, no extension has been made of this technique to continuous systems. In this study, we extend the RGEL technique to nonlinear vibrations of Bernoulli–Euler beams.

2. Regulated Gaussian equivalent linearization

2.1. Derivation of RGEL technique

In Ref. [1], Anh and Di Paola studied the following nonlinear random vibration problem:

$$\ddot{z}+2h\dot{z}+\omega_0^2 z+g(z,\dot{z})=\zeta(t), \tag{1}$$

where $z(t)$, $\dot{z}(t)$, $\ddot{z}(t)$ are the displacement, velocity and acceleration of a single-degree-of-freedom system, respectively; h is the damping coefficient, ω_0 is the natural frequency of the system obtained when $h\equiv 0$, $g\equiv 0$, $\zeta\equiv 0$; $g(z,\dot{z})$ is a nonlinear function, $\zeta(t)$ is a zero-mean Gaussian random excitation. Let $g(z,\dot{z})$ be a polynomial expression of z and $\dot{z}$. The nonlinear function $g(z,\dot{z})$ then takes the following form:

$$g(z,\dot{z})=\sum_{n=0}^{M}\sum_{m=0}^{M}(\alpha_{nm}\dot{z}^{2n}z^{2m+1}+\beta_{nm}z^{2n}\dot{z}^{2m+1}), \tag{2}$$

where α_{nm}, β_{nm} are constants. Since Ref. [1] is not uniformly available, we will describe the method by Anh and Di Paola in some details. First of all, we note that the classical linearization would perform the following replacement of the nonlinear terms by the linear ones:

$$\alpha_{nm}\dot{z}^{2n}z^{2m+1}\to\overline{\alpha}_{nm}z, \tag{3}$$

$$\beta_{nm}z^{2n}\dot{z}^{2m+1}\to\overline{\beta}_{nm}\dot{z}, \tag{4}$$

where the coefficients $\overline{\alpha}_{nm}$, $\overline{\beta}_{nm}$are determined by the minimum mean-square deviation criterion,

$$E[(\alpha_{nm}\dot{z}^{2n}z^{2m+1}-\overline{\alpha}_{nm}z)^2]\to\min_{\overline{\alpha}_{nm}}, \tag{5}$$

$$E[(\beta_{nm}z^{2n}\dot{z}^{2m+1}-\overline{\beta}_{nm}\dot{z})^2]\to\min_{\overline{\beta}_{nm}}. \tag{6}$$

Instead, most unusually, at least at the first glance, Anh and Di Paola [1] suggested to replace nonlinear terms by higher-order nonlinear ones,

$$\alpha_{nm}\dot{z}^{2n}z^{2m+1}\to\alpha_{nm}^{(1)}(\dot{z}^{2n}z^{2m+1})(\dot{z}^{2n}z^{2m})=\alpha_{nm}^{(1)}(\dot{z}^{4n}z^{4m+1}), \tag{7}$$

$$\beta_{nm}z^{2n}\dot{z}^{2m+1}\to\beta_{nm}^{(1)}(z^{2n}\dot{z}^{2m+1})(z^{2n}\dot{z}^{2m})=\beta_{nm}^{(1)}(z^{4n}\dot{z}^{4m+1}), \tag{8}$$

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where the authors used the mean-square criterion for obtaining the coefficients $\alpha_{nm}^{(1)},\ \beta_{nm}^{(1)}$

$$E[(\alpha_{nm}\dot{z}^{2n}z^{2m+1}-\alpha_{nm}^{(1)}\dot{z}^{4n}z^{4m+1})^2]\rightarrow \min_{\alpha_{nm}^{(1)}}, \tag{9}$$

$$E[(\beta_{nm}z^{2n}\dot{z}^{2m+1}-\beta_{nm}^{(1)}z^{4n}\dot{z}^{4m+1})^2]\rightarrow \min_{\beta_{nm}^{(1)}}. \tag{10}$$

The minimization criteria (9) and (10) lead to the following expressions for $\alpha_{nm}^{(1)},\ \beta_{nm}^{(1)}$:

$$\alpha_{nm}^{(1)}=\alpha_{nm}\frac{E[\dot{z}^{6n}z^{6m+2}]}{E[\dot{z}^{8n}z^{8m+2}]}, \tag{11}$$

$$\beta_{nm}^{(1)}=\beta_{nm}\frac{E[z^{6n}\dot{z}^{6m+2}]}{E[z^{8n}\dot{z}^{8m+2}]}. \tag{12}$$

In the next step, Anh and Di Paola replaced higher-order nonlinear terms into the original nonlinear terms,

$$\alpha_{nm}^{(1)}\dot{z}^{4n}z^{4m+1}\rightarrow \alpha_{nm}^{(2)}\dot{z}^{2n}z^{2m+1}, \tag{13}$$

$$\beta_{nm}^{(1)}z^{4n}\dot{z}^{4m+1}\rightarrow \beta_{nm}^{(2)}z^{2n}\dot{z}^{2m+1}, \tag{14}$$

where

$$\alpha_{nm}^{(2)}=\alpha_{nm}^{(1)}\frac{E[\dot{z}^{6n}z^{6m+2}]}{E[\dot{z}^{4n}z^{4m+2}]}, \tag{15}$$

$$\beta_{nm}^{(2)}=\beta_{nm}^{(1)}\frac{E[z^{6n}\dot{z}^{6m+2}]}{E[z^{4n}\dot{z}^{4m+2}]}. \tag{16}$$

The final step is the conventional linear replacement

$$\alpha_{nm}^{(2)}\dot{z}^{2n}z^{2m+1}\rightarrow \alpha_{nm}^{(3)}z, \tag{17}$$

$$\beta_{nm}^{(2)}z^{2n}\dot{z}^{2m+1}\beta_{nm}^{(3)}\dot{z}, \tag{18}$$

where

$$\alpha_{nm}^{(3)}=\alpha_{nm}^{(2)}\frac{E[\dot{z}^{2n}z^{2m+2}]}{E[z^2]}, \tag{19}$$

$$\beta_{nm}^{(3)}=\beta_{nm}^{(2)}\frac{E[z^{2n}\dot{z}^{2m+2}]}{E[\dot{z}^2]}. \tag{20}$$

By substituting the expressions (11) and (15) into (19), the expressions (12) and (16) into (20), we obtain the following expressions for the coefficients $\alpha_{nm}^{(3)},\ \beta_{nm}^{(3)}$:

$$\alpha_{nm}^{(3)}=\alpha_{nm}\frac{E[\dot{z}^{2n}z^{2m+2}]E[\dot{z}^{6n}z^{6m+2}]E[\dot{z}^{6n}z^{6m+2}]}{E[z^2]\quad E[\dot{z}^{4n}z^{4m+2}]E[\dot{z}^{8n}z^{8m+2}]}, \tag{21}$$

$$\beta_{nm}^{(3)}=\beta_{nm}\frac{E[z^{2n}\dot{z}^{2m+2}]E[z^{6n}\dot{z}^{6m+2}]E[z^{6n}\dot{z}^{6m+2}]}{E[\dot{z}^2]\quad E[z^{4n}\dot{z}^{4m+2}]E[z^{8n}\dot{z}^{8m+2}]}. \tag{22}$$

The results (21) and (22) can be also obtained by using statistical orthogonality of the difference of the left and right hand sides in Eqs. (7), (13) and (17) and Eqs. (8), (14) and (18) as presented in [2]. The linearized equation of the original nonlinear Eq. (1) takes the following form:

$$\ddot{z}+2h\dot{z}+\omega_0^2 z+\sum_{n=0}^{M}\sum_{m=0}^{M}(\alpha_{nm}^{(3)}z+\beta_{nm}^{(3)}\dot{z})=\zeta(t), \tag{23}$$

where $\alpha_{nm}^{(3)},\ \beta_{nm}^{(3)}$ are found from (21) and (22).

In order to elucidate the RGEL technique, as shown in [1], the authors evaluated several oscillators under white noise excitation. For illustration, in the following subsection, responses of a Duffing oscillator subjected to random external force are considered.

2.2. Response of Duffing oscillator

Consider the following Duffing oscillator subjected to random excitation:

$$\ddot{z}+2h\dot{z}+\omega_0^2 z+\gamma z^3=\zeta(t), \tag{24}$$

where $h,\ \omega_0,\ \gamma$ are positive real constants, $\zeta(t)$ is a zero-mean Gaussian white noise excitation with the constant spectral density S_0 and correlation $R_\zeta(t)$

$$R_\zeta(\tau)=E[\zeta(t)\zeta(t+\tau)]=2\pi S_0\delta(\tau) \tag{25}$$

where $\delta(t)$ is the Dirac delta function. It is noted that, to obtain the Duffing system (24), the parameters of Eq. (1) are taken to be $M=1,\ \ n=0,\ \ m=1,\ \ \alpha_{00}=0,\ \ \alpha_{01}=\gamma,\ \beta_{00}=\beta_{01}=0$. The linearized equation of Eq. (24) takes the following form:

$$\ddot{z}+2h\dot{z}+\omega_0^2 k_{eq}z=\zeta(t), \tag{26}$$

where the non-dimensional linearization coefficient $k_{eq}$is found from a specified criterion of the linearization method. Here, we use the regulation linearization procedure as presented above for obtaining the coefficient k_{eq}. For this purpose, a linearization scheme for the nonlinear term γz^3 of the original Eq. (1) is applied (see also in [2]),

$$\gamma z^3\rightarrow\frac{\gamma}{9E[z^2]}z^5\rightarrow\frac{7\gamma}{9}z^3\rightarrow\frac{7\gamma}{3}E\left[z^2\right]z. \tag{27}$$

The regulation process for the nonlinear term γz^3 is taken according to the replacement steps (7), (13), (17). The linearization coefficient k_{eq} is found to be

$$k_{eq}=1+\frac{7\gamma}{3\omega_0^2}E\left[z^2\right]. \tag{28}$$

In order to find an approximate expression of $E[z^2]$, we utilize the following relationship for the linearized Eq. (26) between the mean-square response $E[z^2]$ and spectral density S_0 of random excitation (see [4] for details):

$$E\left[z^2\right]=\frac{\pi S_0}{2h\omega_0^2 k_{eq}}. \tag{29}$$

Substituting Eq. (29) into Eq. (28) yields the following equation for the unknown k_{eq}:

$$k_{eq}=1+\frac{7\pi\gamma S_0}{6h\omega_0^4 k_{eq}}. \tag{30}$$

By solving Eq. (30) for the unknown k_{eq}, we obtain

$$k_{eq}=\frac{1}{2}\left(1+\sqrt{1+\frac{14\pi\gamma S_0}{3h\omega_0^4}}\right). \tag{31}$$

Substituting the result (31) into the right hand side of Eq. (29), we get an approximate expression for determining the mean-square response $E[z^2]$

$$E[z^2]_{\text{regulated}}=\frac{\pi S_0}{h\omega_0^2}\frac{1}{1+\sqrt{1+\frac{14\pi\gamma S_0}{3h\omega_0^4}}}. \tag{32}$$

Similarly, for the case of conventional linearization, one gets the expression of mean-square response $E[z^2]$

$$E[z^2]_{\text{conventional}}=\frac{\pi S_0}{h\omega_0^2}\frac{1}{1+\sqrt{1+\frac{6\pi\gamma S_0}{h\omega_0^4}}}. \tag{33}$$

The exact solution of $E[z^2]$ corresponding to the original nonlinear Eq. (24) is evaluated by [5]

$$E[z^2]_{\text{exact}}=\frac{\int_{-\infty}^{\infty}z^2\exp\left\{-\frac{2h}{\pi S_0}\left(\frac{1}{2}\omega_0^2 z^2+\frac{1}{4}\gamma z^4\right)\right\}dz}{\int_{-\infty}^{\infty}\exp\left\{-\frac{2h}{\pi S_0}\left(\frac{1}{2}\omega_0^2 z^2+\frac{1}{4}\gamma z^4\right)\right\}dz}. \tag{34}$$